GLIMMPSE User Manual

Version 2.0.0

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Contents

1. Introduction

1.1. Version Information and Licensing

This manual describes version 2.0.0 of the GLIMMPSE software. The manual applies to all 2.0.x versions of GLIMMPSE (e.g. 2.0.0, 2.0.1, 2.0.2, etc.).

1.2. Welcome to GLIMMPSE 2.0.0

GLIMMPSE 2.0.0 is an open-source online tool for calculating power and sample size. GLIMMPSE has been designed so that researchers and scientists with a varying levels of statistical training can have access to reliable power and sample size calculations. For optimum usability, GLIMMPSE provides two different modes. In *Guided Mode* users receive step-by-step guided instructions for entering data in order to obtain power and sample size outputs. In *Matrix Mode* users receive less guidance, and are assumed to possess in-depth statistical training.

GLIMMPSE can compute power or sample size for univariate and multivariate linear models with Gaussian errors (Muller and Stewart 2006). GLIMMPSE supports two main types of study design models: designs with only fixed predictors, and designs with fixed predictors and a single Gaussian covariate. The values of a fixed predictor are set as part of the study design, and are known without appreciable error. In contrast, Gaussian covariates are not observed until data is collected. Common designs with only fixed predictors include t-tests, analysis of variance (ANOVA), and multivariate analysis of variance (MANOVA). Common designs that control for a covariate include analysis of covariance (ANCOVA) and multivariate analysis of covariance (MANCOVA).

Details about power calculations for the general linear multivariate model with Gaussian data and fixed predictors can be found in Muller and Peterson (1984), Muller and Barton (1989), Muller, Lavange, Ramey, and Ramey (1992), Muller, Edwards, Simpson, and Taylor (2007), Muller and Stewart (2006), and Muller *et al.* (2007). Details for fixed predictors with a single Gaussian covariate can be found in Glueck and Muller (2003).

GLIMMPSE utilizes a Java web services architecture (McGovern, Tyagi, Stevens, and Mathew 2003), designed to facilitate future support of additional statistical models. The tool is hosted at `http://glimmpse.samplesizeshop.org`.

1.3. Why GLIMMPSE?

Other programs, such as POWERLIB, NQuery, and Pass, also calculate power and sample size. So why use GLIMMPSE?

GLIMMPSE has several advantages over these other programs, because GLIMMPSE:

1. **Is free**. GLIMMPSE provides free online power and sample size computing.
2. **Is user friendly**. In both *Guided Mode* and *Matrix Mode* GLIMMPSE provides a step-by-step interface to assist researchers in producing accurate power and sample size calculations.

3. Calculates power and sample size for any univariate or multivariate test for the general linear multivariate model, assuming fixed predictors.
4. Produces confidence intervals on power estimates for designs with fixed predictors.
5. Produces power and sample size calculations for designs with a single Gaussian covariate.
6. Supports designs with unequal group sizes, and complicated covariance structures.
7. Creates basic power curves.

2. Using GLIMMPSE

2.1. When to Use GLIMMPSE

GLIMMPSE is a tool researchers and scientists can use to calculate reliable values for power and sample size. GLIMMPSE calculates power or sample size for designs with normally distributed outcomes, and for a variety of multilevel and longitudinal studies. GLIMMPSE can calculate power and sample size for common statistical tests and models including:

- One sample t-test
- Paired t-test
- Two sample t-test
- Analysis of variance (ANOVA)
- Analysis of covariance (ANCOVA)
- Repeated measures analysis of variance
- Multivariate analysis of variance (MANOVA)
- Multivariate analysis of covariance (MANCOVA)

2.2. How to Use GLIMMPSE

2.2.1. Initiating the GLIMMPSE Wizard

GLIMMPSE can be accessed with a standard web browser at `http://glimmpse.samplesizeshop.org/`. The GLIMMPSE start screen is shown in Figure 1. GLIMMPSE has been tested in Internet Explorer 8 (Microsoft 2010), Mozilla Firefox 13.0.1 (Mozilla 2011), Google Chrome 23.0.1271.95 (Google 2011) and Safari 5.0.3 (Apple 2010).

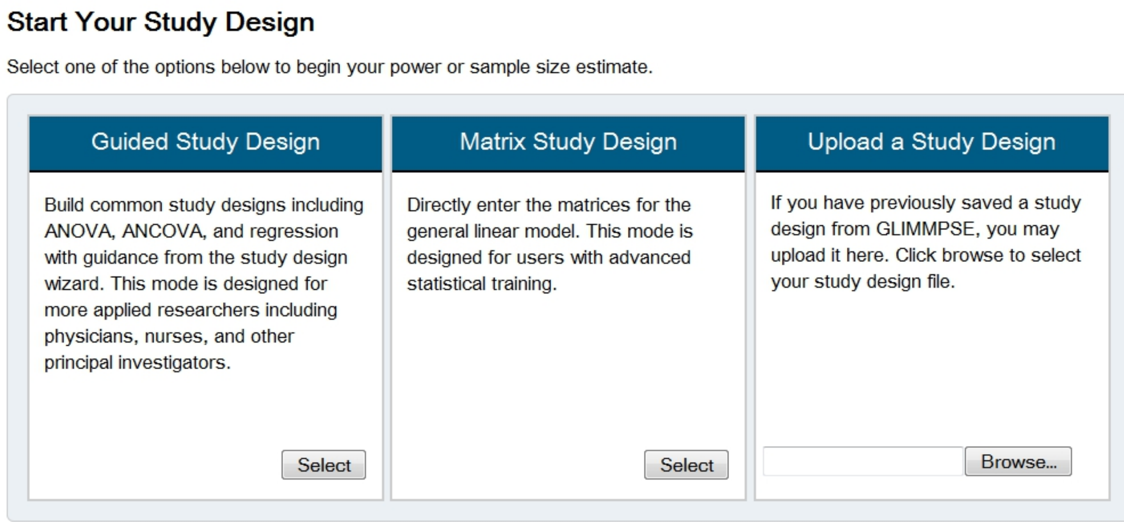

Figure 1: GLIMMPSE Start Screen

2.2.2. Choosing Between Guided Mode and Matrix Mode

The GLIMMPSE start screen presents three options: *Guided Mode*, *Matrix Mode*, and *Upload a Study Design*.

In *Guided Mode* users receive step-by-step guided instructions when entering inputs for power or sample size calculations. To choose Guided Mode, click Select in the Guided Study Design box.

In *Matrix Mode* users receive less guidance, and are assumed to possess in-depth statistical training. *Matrix Mode* allows direct input of all matrices required for a power or sample size calculation. To choose Matrix Mode, click Select in the Matrix Study Design box.

If the user has a study design saved from a previous GLIMMPSE session, the user may upload it by clicking Choose File in the *Upload a Study Design* box. GLIMMPSE will open the saved study design and allow the user to continue the power or sample size analysis.

2.3. Basic Navigation for GLIMMPSE in Both *Guided Mode* and *Matrix Mode*

Once a mode of entry has been chosen, the steps required for GLIMMPSE to calculate power are listed as tabs on the left side of the *Introduction* screen. A white background indicates a tab as active, and a blue background designates a tab as inactive. Black text designates a page within a tab as active, and gray designates a page as inactive. Only one page within one tab can be active at a time.

On the bottom right of any screen in GLIMMPSE is a menu of options enabling users to save their study design by clicking Save Design, consult the help library by clicking Help, or cancel without saving and return to the *Start Your Study Design* screen by clicking Cancel.

Each section is broken into one or more sub-sections with the title in bold at the top of the page. Each screen contains instructions and/or areas for user inputs. Users navigate through the sections and sub-sections by clicking to advance or to go back. The user may also navigate by clicking on the section titles in the left navigation panel.

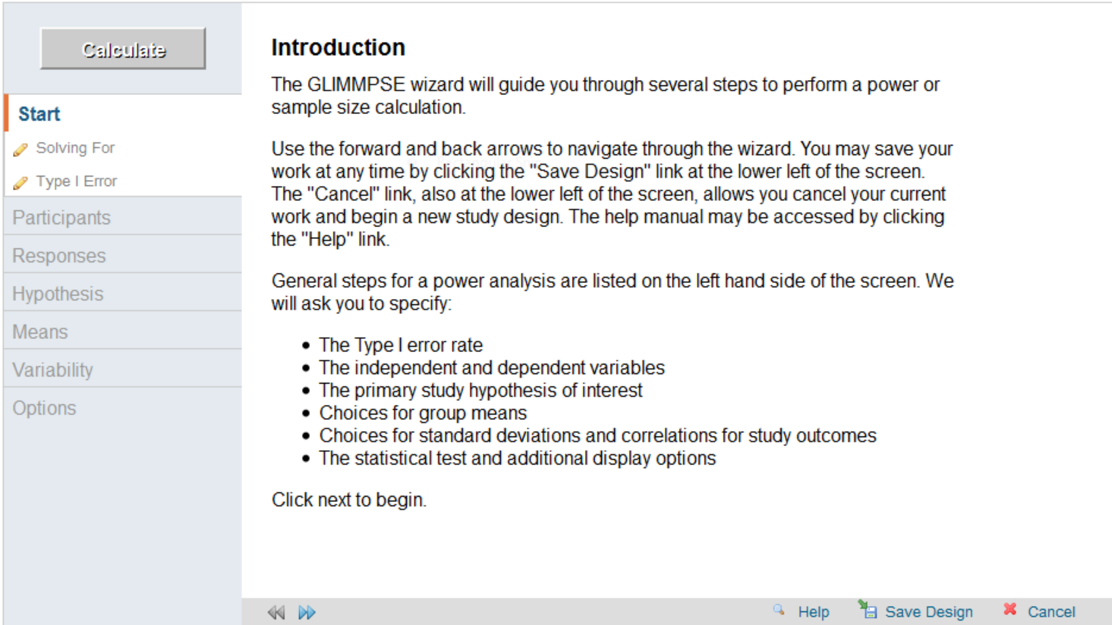

Figure 2: Example Start Screen

Following the *Introduction* screen (Figure 2), GLIMMPSE will prompt the user to enter the details for the power or sample size calculation. The user may enter the details in any order. However, some screens cannot be accessed unless the user has completed information from previous screens. For example, if the user has not entered information in the *Responses* tab, then the user will not be able to enter information in the *Hypothesis* tab. If a tab is inaccessible due to missing information in an earlier screen, it will be indicated by a red circle with a slash through it:

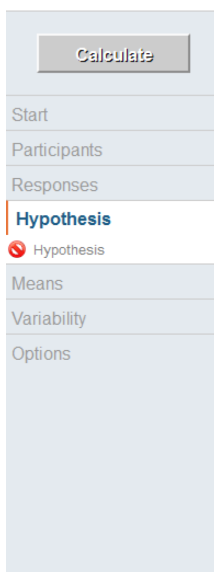

Figure 3: More information is required to access the *Hypothesis* screen.

Notice that in Figure 2 above there are pencil icons beside the two available screens in the active *Start* tab.

The pencil icons indicate screens which require additional information. Once the user have entered the required information, the pencil will turn into a green check mark:

Figure 4: Indication that the *Solving for* screen is complete.

Some screens are optional, and will already have a check mark beside them.

2.3.1. Typing Into a Text Box

Several screens in GLIMMPSE will ask you to specify information by typing into a text box. To input information in a text box, click in the text box and type the requested information. To complete the entry you may: 1) Click anywhere on the screen; 2) Press Enter on your keyboard; or, 3) Click Add for text boxes associated with a list of values.

To delete entries in a list associated with the text box, click on the entry so that it becomes highlighted in blue. Click Delete to delete the highlighted entry.

Figure 5 shows three examples of text box entries with the text boxes highlighted in yellow.

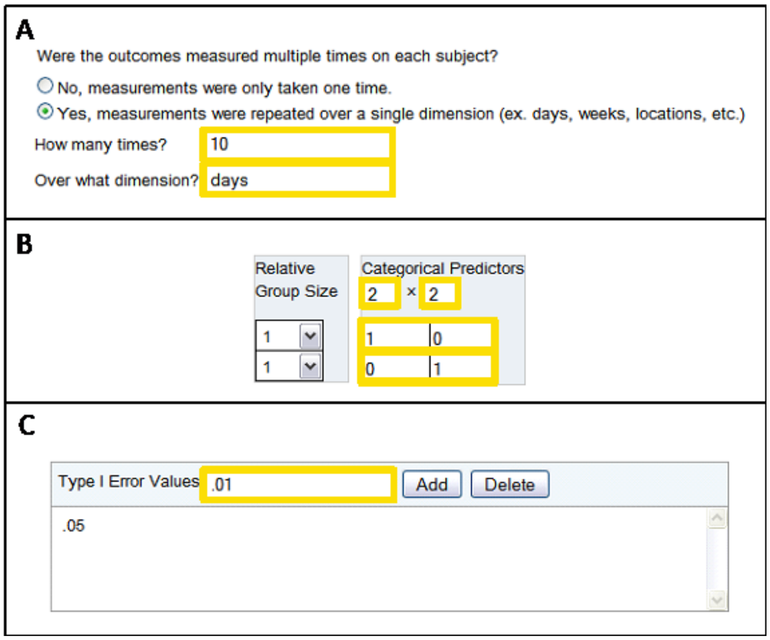

Figure 5: Examples of text boxes that are used to A) collect information on repeated measures; B) specify the size and contents of a matrix; and C) specify one or more choices for an item used in the power calculation.

2.3.2. *Using Drop Down Lists*

When GLIMMPSE requires you to choose from a defined list of options these options will be presented in a drop down list. Figure 3 shows an example of a drop down list. To choose an option from a drop down list, click on the down arrow (see [1]), then select your choice from the list of options (see [2]).

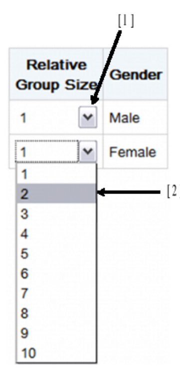

Figure 6: Example of a drop down menu.

2.3.3. *Radio Buttons and Check Boxes*

In some cases, you must choose from a list of options by selecting a radio button or checking a box. The radio buttons allow you to select only one option. The check boxes allow you to select more than one option. To select an option, click on the radio button or check the box next to that option. Figure 7 shows an example of a radio button (see A), and a check box (see B).

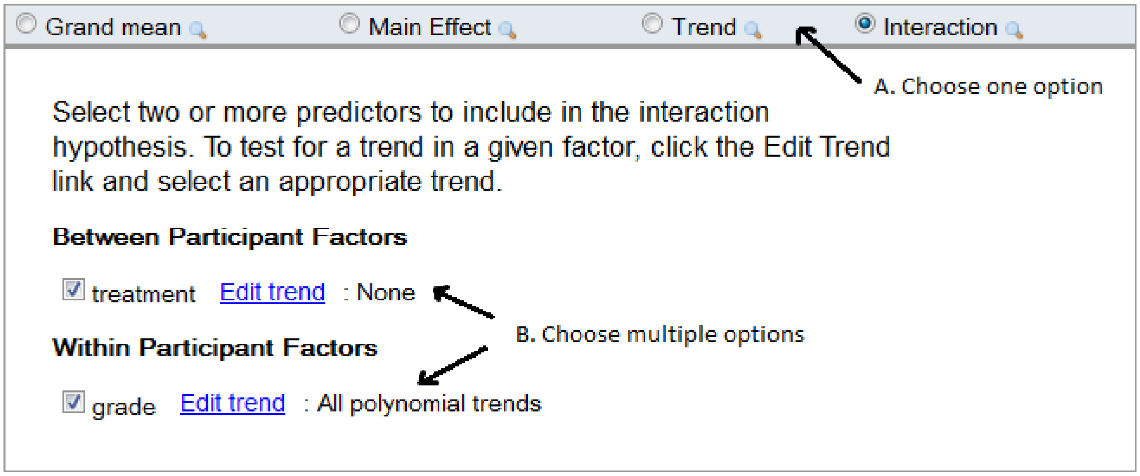

Figure 7: Example of radio buttons and check boxes.

2.3.4. Results Report

Power results are displayed in a table with each row representing an individual power calculation. If multiple factors have been specified in the study design (for example, multiple Type I error rates, variability scale factors, etc.) then the results table will have multiple rows. See Table 1 below for an example of the information displayed for a given results report.

Every results report for power contains both calculated and desired power values. When solving for power, these two values are the same. When solving for sample size, it may not be possible to achieve the exact power value specified by the user. In this case, *nominal power* is the default power value (the power value specified by the user), and *actual power* is the calculated power for the sample size that best matches the desired power.

A power curve may also be requested, with power on the vertical, or Y, axis and either the regression coefficient scale factor, covariance scale factor, or total sample size on the horizontal, or X, axis.

Power results can be saved to a comma delimited file so that users can import the data into other statistical packages. To save the power results, click Save to CSV beneath the table of results. For transparency, the matrices used in the calculations are accessible on the results screen. To view the exact matrices used in the calculations, click View Matrices beneath the table of results. This is most useful in *Guided Mode*, where matrix information is largely hidden from the user.

Column Name	Description
Test	Name of the statistical test
Actual Power	Calculated power
Total Sample Size	Total number of research participants required to achieve the actual power
Beta Scale	Scale factor applied to the $\mathbf{B}$ or $\mathbf{B}_F$ matrix
Sigma Scale	Scale factor applied to the $\mathbf{\Sigma}_E$ matrix
Alpha	The Type I error value
Nominal Power	The desired power
Power Method	Indicates whether conditional, unconditional, or quantile power was used
Quantile	If the current power method is quantile power, this indicates the quantile of the distribution of possible powers. Otherwise, this field is empty.
Power Lower	Lower limit of the 95% confidence interval
Power Upper	Upper limit of the 95% confidence interval

Table 1: Information displayed for each power result.

2.4. Basic Navigation for *Guided Mode*

2.4.1. Entering Predictor Variables

In Guided Mode, GLIMMPSE requires the user to enter labels for predictor variable(s) (also called independent variables) and outcome variable(s) (also called dependent variables). Figure 8 shows an example of entering variable labels. To enter a variable label, type the label into the text box provided (see [1] in Figure 8). After each entry, click Add, press Enter on the keyboard, or click anywhere on the screen to populate the field below that text box (see [2] in Figure 8).

For the predictor variables, GLIMMPSE also asks the user to specify the categories for each variable. For example, the predictor variable "gender" has two categories, "male" and "female." To specify categories associated with a given predictor, select a predictor in the text box on the left (see [2]), then enter the category labels into the text box on the right (see [3] in Figure 8). After each entry, click Add, press Enter on the keyboard, or click anywhere on the screen to populate the category text box (see [4]).

Only category labels associated with the highlighted predictor label are shown. To delete predictors or category labels, select the unwanted label and click Delete. This removes the label from the list. If the user removes a predictor, the associated categories are automatically deleted.

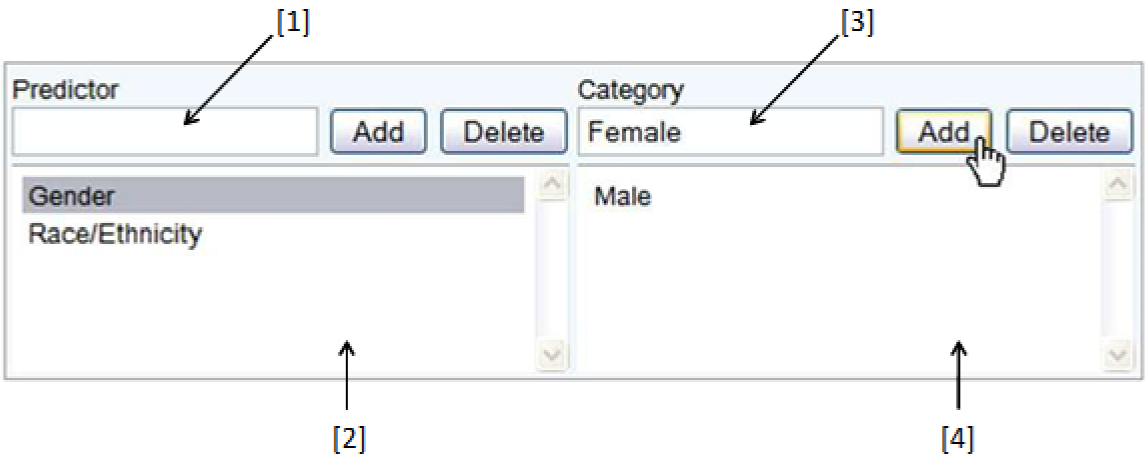

Figure 8: Example of entering labels

2.5. Basic Navigation for *Matrix Mode*

2.5.1. Resizing and Entering Values Into a Matrix

In Matrix Mode, GLIMMPSE requires the user to define the matrices for the power calculation. Figure 9 shows an example of a matrix template in *Matrix Mode*. Sometimes the matrix dimensions are pre-determined. If not, the user can set the matrix dimensions by typing the number of rows into the row text box (see [1] in Figure 9) and the number of columns into the column text box (see [2] in Figure 9). Fill in the elements of the matrix by entering values into the text boxes within the matrix template (see [3] in Figure 9).

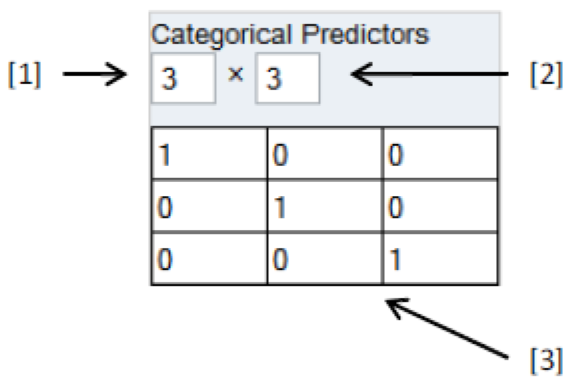

Figure 9: Example of entering values into a matrix

3. Using *Guided Mode: A Screen-by-Screen Tour*

In *Guided Mode* users receive step-by-step guided instructions when entering inputs for calculating power and

sample size for use in study design.

3.1. Start

3.1.1. Introduction

The *Introduction* screen contains a summary of the steps involved in the power or sample size analysis. After reading the screen, click to begin entering the details of the study design.

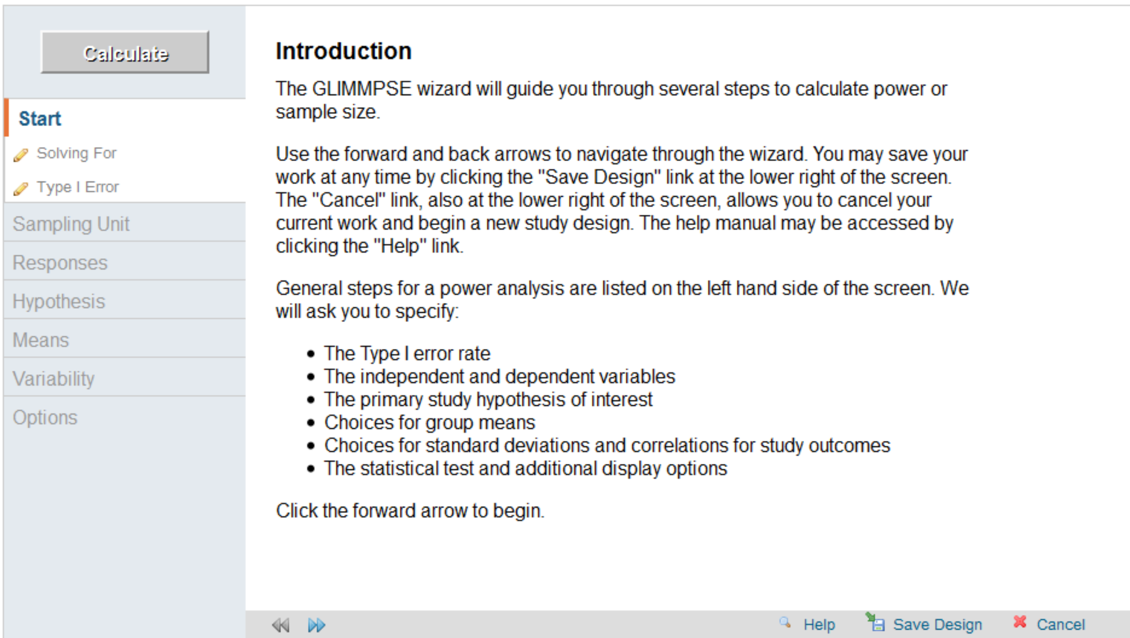

Figure 10: Introduction Screen

3.1.2. Solving For?

The *Solving For?* screen allows the user to select either a power or sample size calculation.

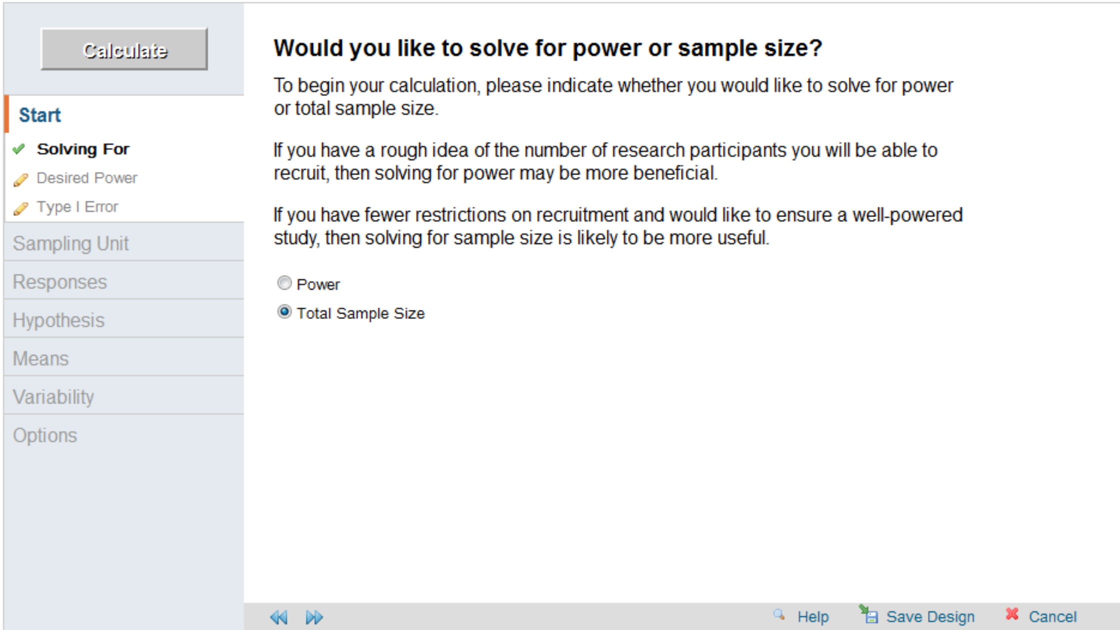

Figure 11: Solving For? Screen

When *power* is selected, the inputs will be used for a power analysis. The power analysis will produce a value(s) between 0 and 1, representing the probability the study will provide an answer to the question of interest. When *sample size* is selected, the inputs will be used to calculate the number of individual sampling units (also called participants, if referring specifically to people) needed for the study to achieve the desired power.

If the number of participants is not set, we recommend solving for sample size in order to obtain the appropriate sample size for achieving the goals of your study. However, if the sample size is set due to budgetary or other restrictions, a power calculation will indicate the probability that your study will provide a definitive answer to the question of interest.

On the screen, select *Power* or *Total Sample Size* by selecting the appropriate radio button.

Upon completing the selection, click to proceed.

3.1.3. Desired Power (if solving for Total Sample Size)

When solving for sample size, the user must enter the desired power for the study. Enter the target values as decimals, i.e. 0.95, in the Power Values box and click Add to add the value to the list.

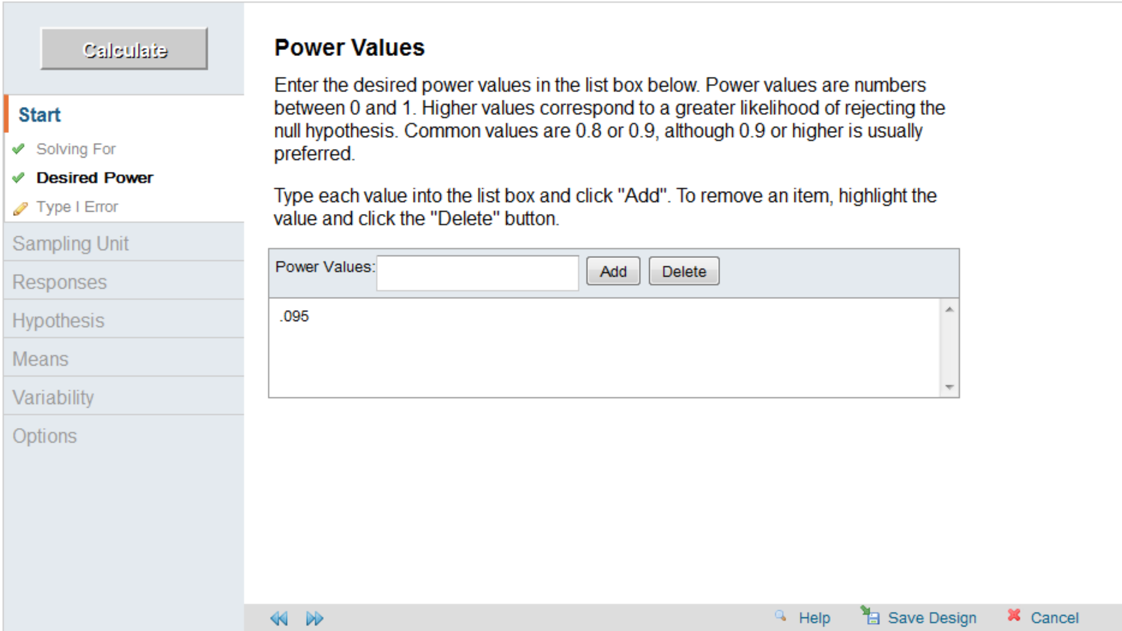

Figure 12: Desired Power Screen

When finished, click to proceed.

3.1.4. Type I Error

Enter the target values for Type I Error as decimals (i.e. 0.05) in the Type I Error Values box. The user may specify up to five Type I Error values.

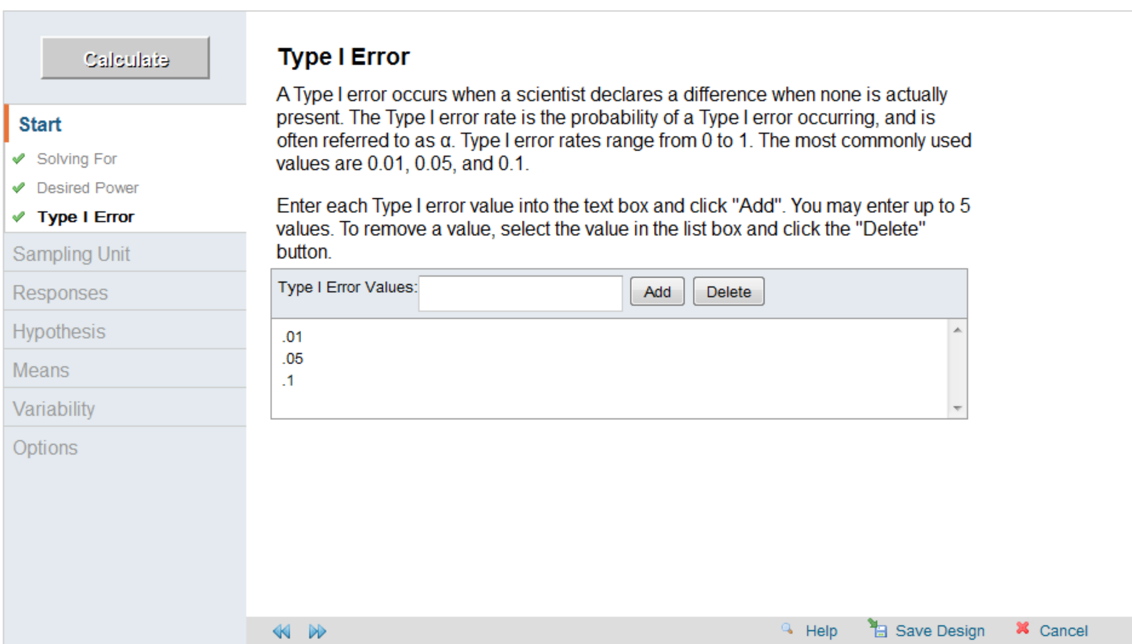

Figure 13: Type I Error

When finished, click to proceed.

3.2. Sampling Units

3.2.1. Introduction

This screen provides an introduction to the *Sampling Units* section and defines the concept of an independent sampling unit. The sampling unit is typically the study participant. For multilevel designs and cluster randomized trials, the sampling unit may be a group of participants, such as schools or neighborhoods.

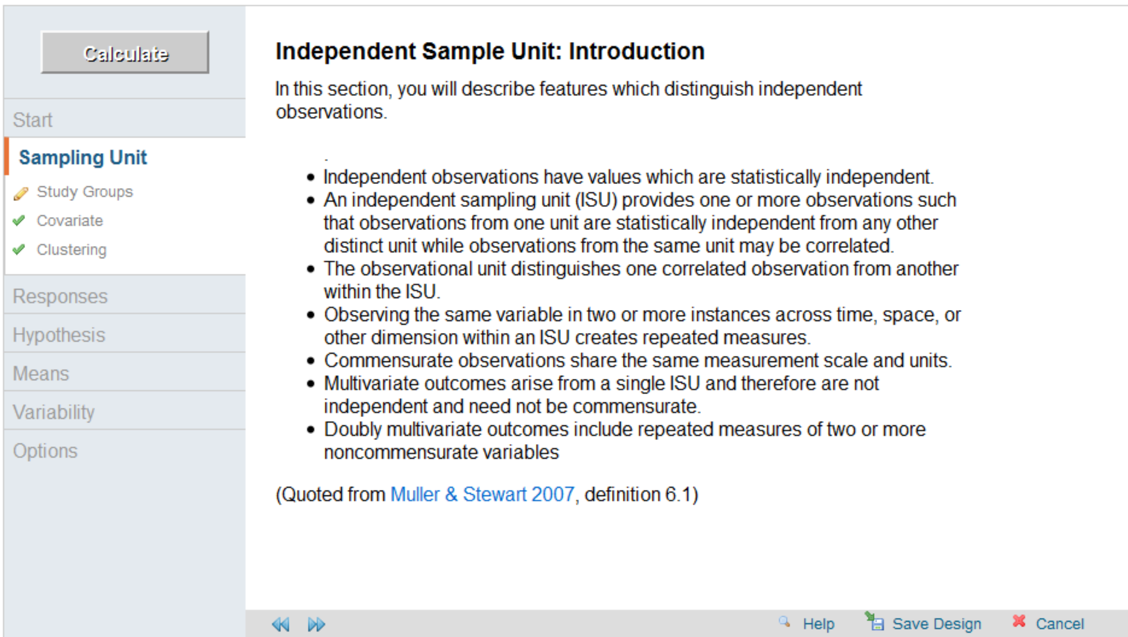

Figure 14: Sampling Units: Introduction Screen

After reading the information on the screen, click to proceed.

3.2.2. Study Groups

Independent sampling units may be randomized to different treatments, or be classified by characteristics such as gender. The characteristics divide the sampling units into study groups. The *Study Groups* screen allows the user to define the study groups by specifying *fixed predictors*. Enter the fixed predictors as described in Section 2.4.1. For one-sample designs with no fixed predictors, leave the table blank.

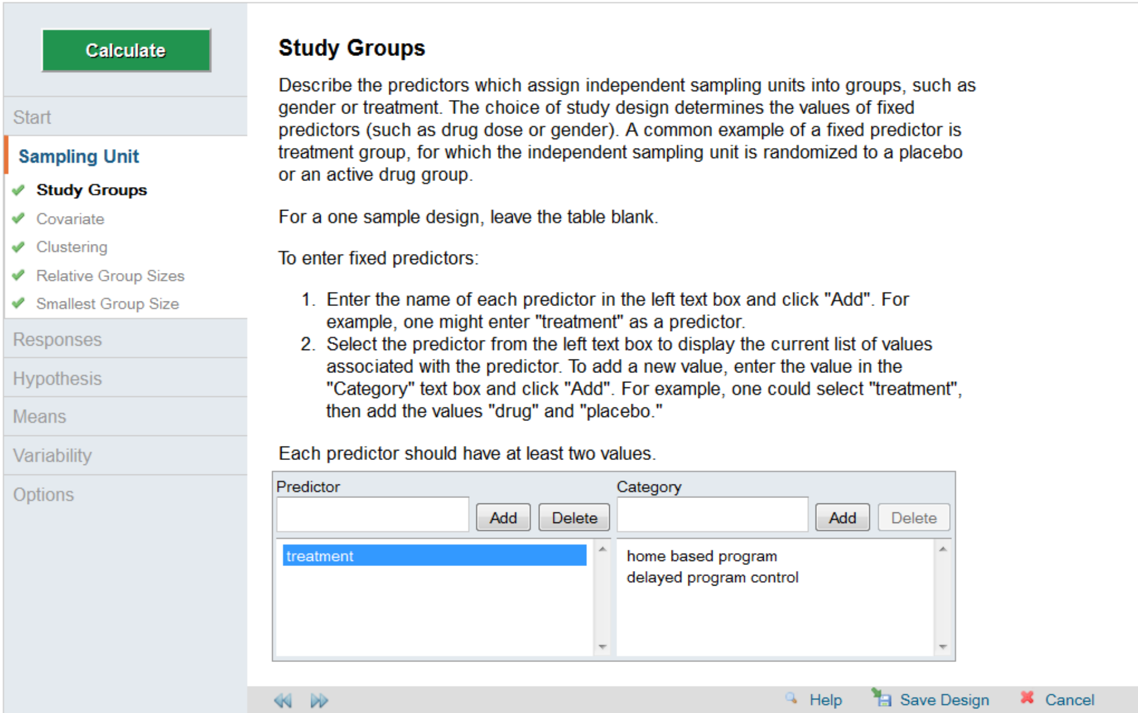

Figure 15: Study Groups Screen

When finished, click ⏩ to proceed.

3.2.3. Covariate

The *Covariate* screen allows the user to control for a single, normally distributed predictor (also known as a normally distributed covariate). For example, a scientist may wish to examine the effect of a drug when controlling for age. In this case, age would be the covariate. If the study design does not include a normally distributed predictor, leave the checkbox blank. If the study design does include a covariate, check the checkbox.

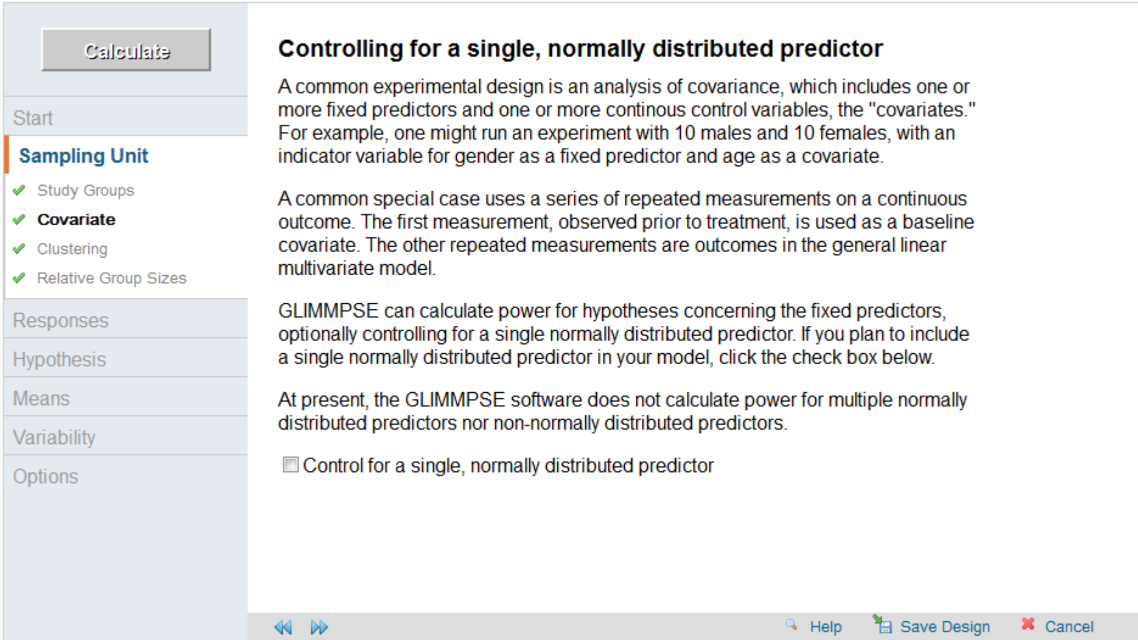

Figure 16: Gaussian Predictor Screen

When finished, click to proceed.

3.2.4. Clustering

Clustering is present when research participants are organized into groups. Often, randomization in a study occurs at the group level rather than by individual research participants. The *Clustering* screen allows the user to enter up to three levels of clustering.

An example of clustering would be a study design in which the participants are students randomly selected from different schools in an area. In this case, each school would represent a cluster. An example of subgroups within a cluster would be each classroom within a given school.

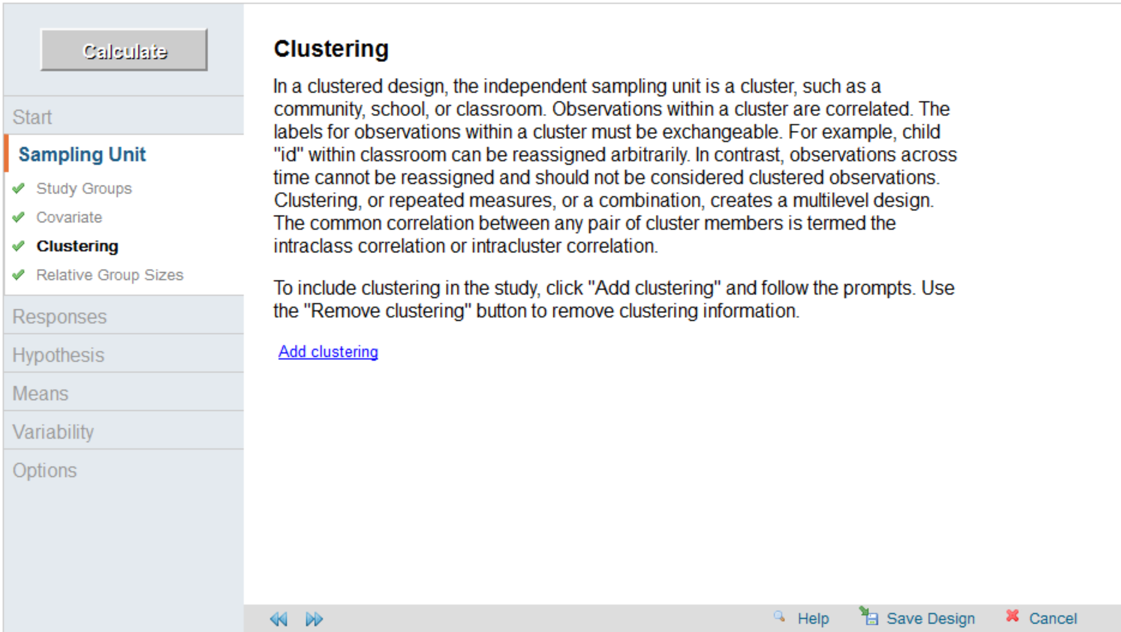

Figure 17: Clustering Screen

If the study design does not include clustering, simply click to proceed.

To add clustering, click the *Add clustering* button. Three text boxes will appear at the bottom of the screen:

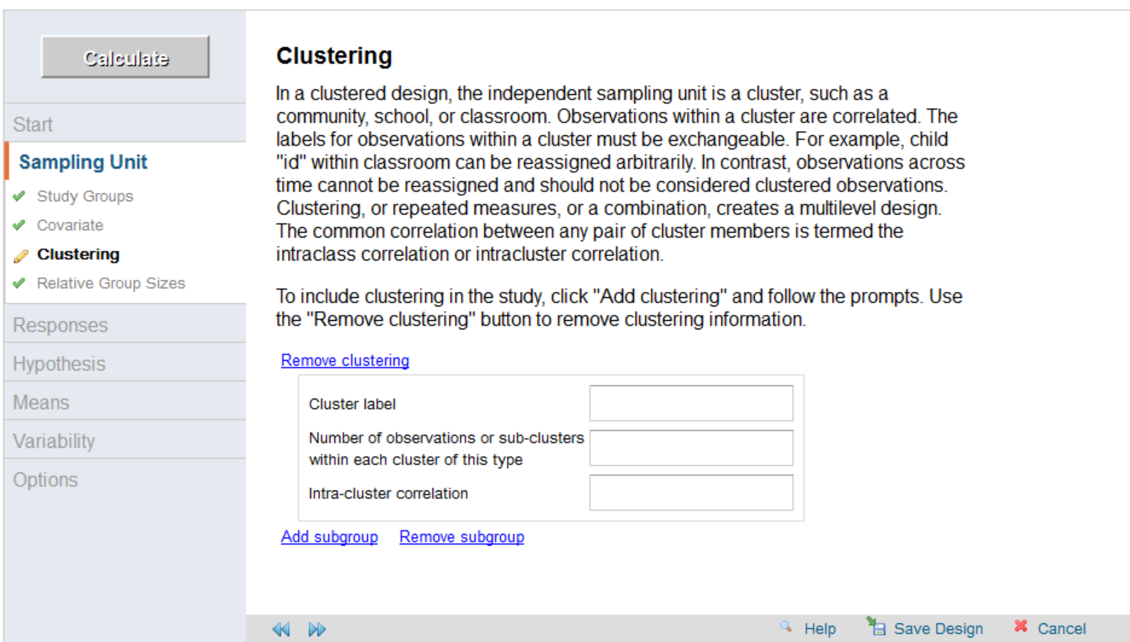

Figure 18: Clustering Options

Enter the *Cluster name*, specify the *Number of observations or sub-clusters within each cluster of this type*, and specify the *Intra-cluster correlation*. The Intra-cluster correlation is the expected correlation between pairs of observations within the cluster.

To add a subgroup to the cluster, click *Add subgroup* and fill in the information for that subgroup. GLIMMPSE allows one primary cluster and two subgroups.

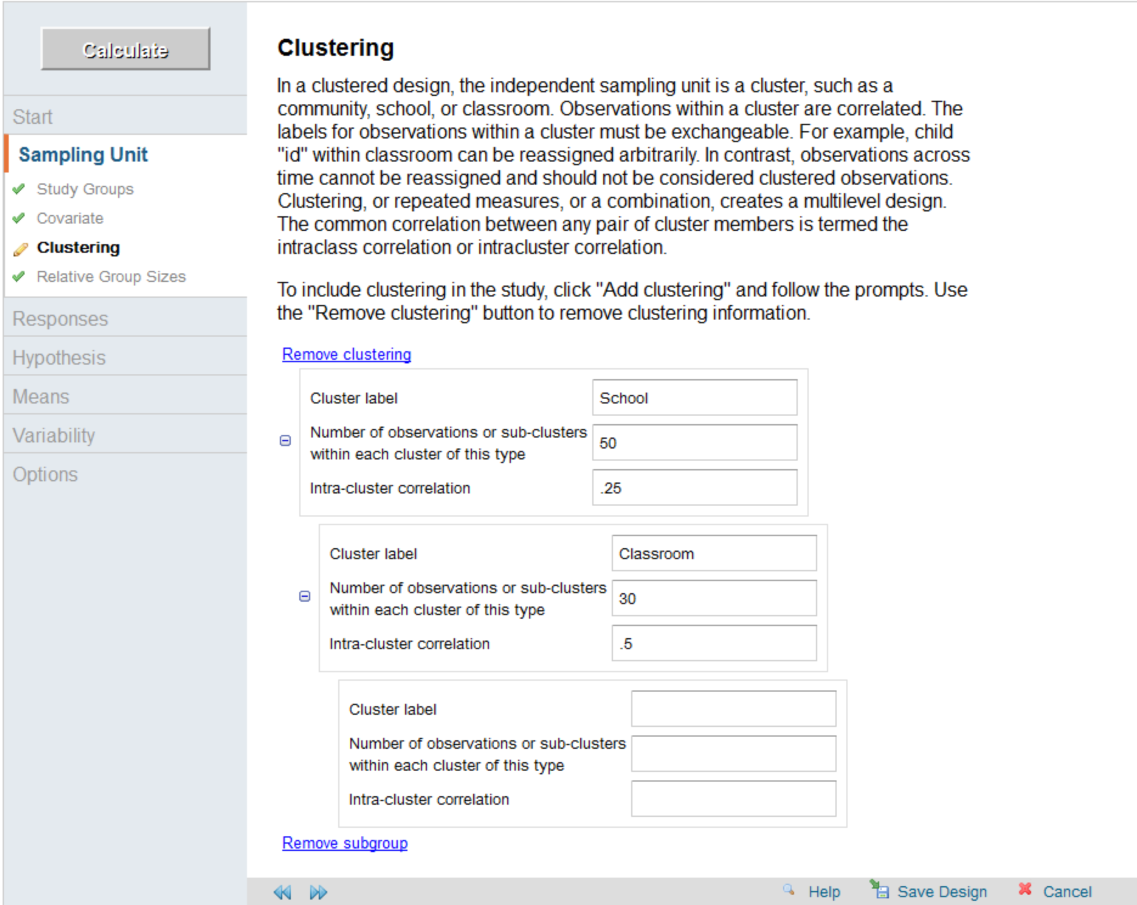

Figure 19: Clustering Sub-Groups

Continuing with the above example, the subgroup *Cluster name* would be "classroom," the *Number of observations* would be the number of students within each classroom, and the *Intra-cluster correlation* would be the expected agreement between students within each classroom.

To remove a subgroup or remove clustering, simply click *Remove subgroup* or *Remove clustering*.

When finished, click ⏩ to proceed.

3.2.5. Relative Group Sizes (if solving for Power)

For designs with multiple study groups (see Section 3.2.2), the user may specify equal or unequal group sizes. On the Relative Group Sizes screen, the user can select the relative sizes of each group by selecting a value from the drop down list.

For example, consider a design with males and females, randomized to receive either an active drug or a placebo. For equal group sizes, a "1" should be selected for each drop down list as shown in Figure 20. However, if there were twice as many males receiving the drug compared to females receiving the drug, the user would select a "2" for the male + drug group, and "1" for the remaining groups.

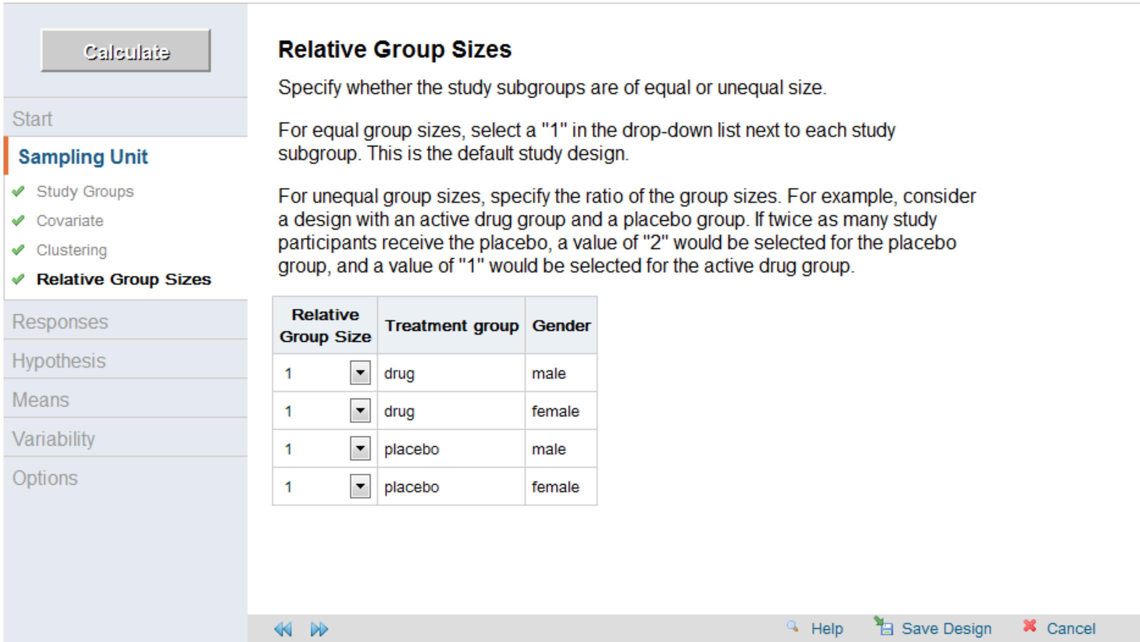

Figure 20: Relative Group Sizes Screen

When finished, click to proceed.

3.3. Smallest Group Size

When solving for power, the user specifies the total sample size for the design by the relative group sizes and the smallest group size. On the *Smallest Group Size* screen, the user may enter one or more values describing the number of participants in the smallest group.

For example, consider a design with a treatment and a placebo group, in which three times as many participants receive the treatment compared to the placebo. With a smallest group size of 20, 30, or 40, the total sample size for the design would be 80 (i.e. 60 treated participants, 20 with placebo), 120, and 160 participants respectively.

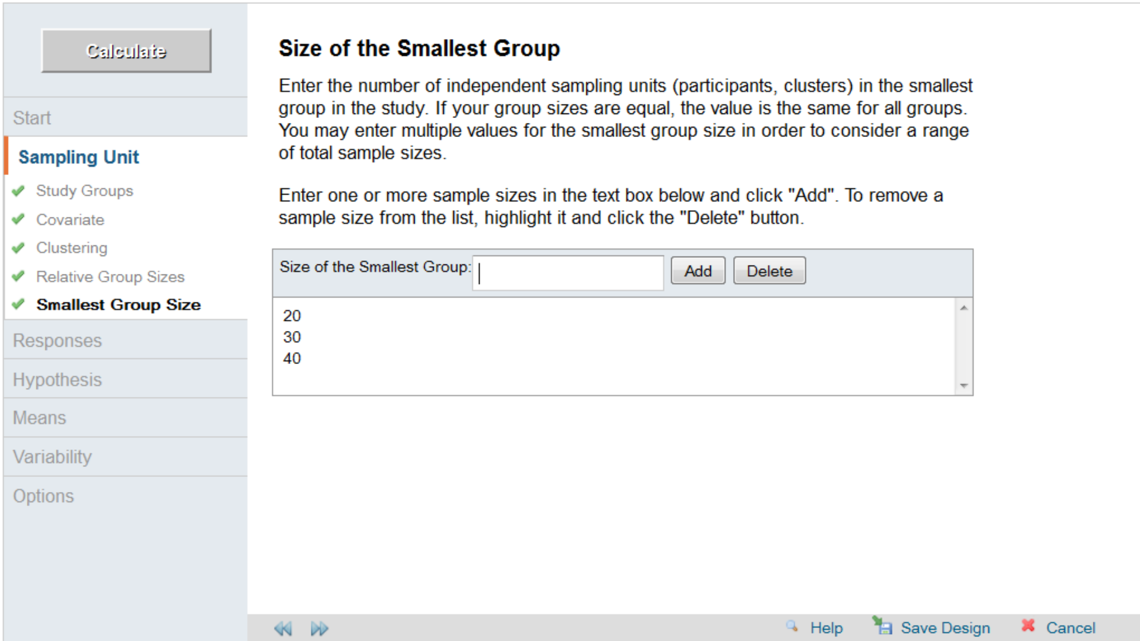

Figure 21: Smallest Group Size Screen

When finished, click to proceed.

3.4. Responses

3.4.1. Introduction

This screen provides an introduction to the *Responses* section. After reading the information on the screen, click to proceed.

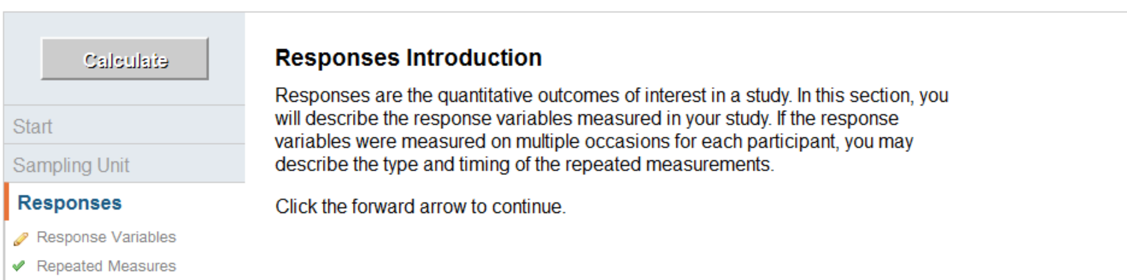

Figure 22: Responses Introduction Screen

3.4.2. Response Variables

The *Response Variables* screen allows the user to specify the response or dependent variables for the study. For example, if "expected pain" is the desired outcome, enter "expected pain" in the text box.

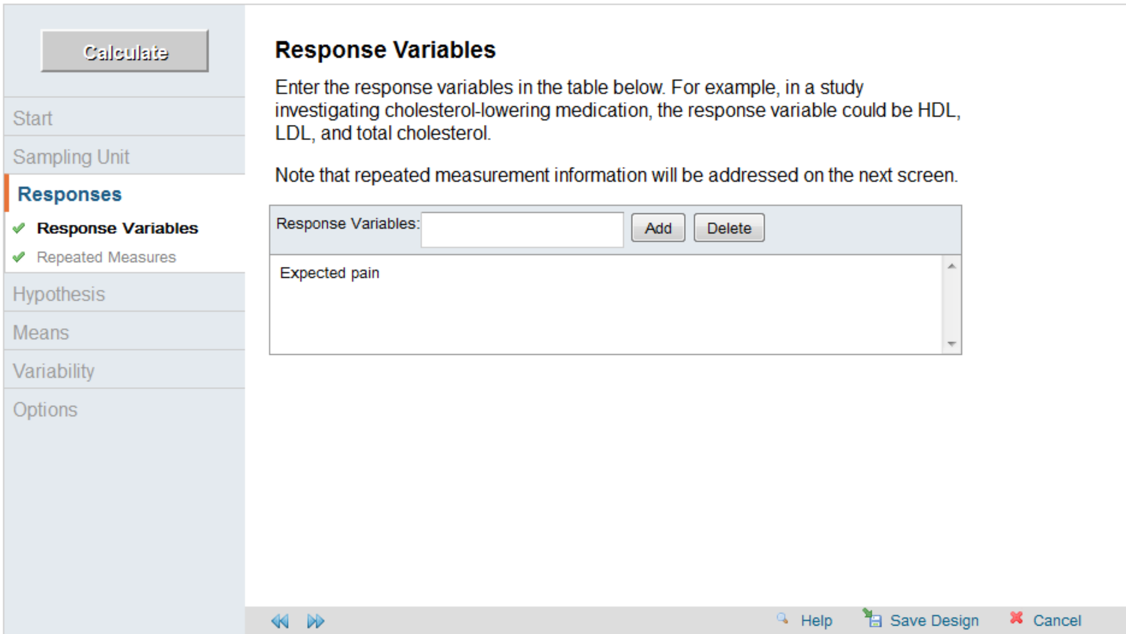

Figure 23: Response Variables

When finished, click to proceed.

3.4.3. Repeated Measures

The *Repeated Measures* screen allows the user to describe repeated measures. Repeated measures are present in a study when multiple measurements are taken on each research participant. An example of repeated measures would be researchers taking a participant's blood pressure once a month for six months.

If the design does not have repeated measures, click to proceed. If the design includes repeated measures, click *Add repeated measures* and fill in the requested information.

Units is a user-specified description of the repeated measure. For example, if the repeated measures are taken once every month, the unit could be "month." Enter a label for the units of the repeated measure.

Enter the *Type* of unit. For *Numeric* repeated measures, both the distance and ordering between measurements is meaningful. Measuring blood pressure every month for 6 months is a numeric repeated measure. GLIMMPSE will auto-populate an equal distance between repeated numeric measures. You can change the distance between the measures by typing into the text boxes. For *Ordinal* repeated measures, the ordering of the measurements is meaningful, but the distance between measurements is assumed to be equal. For example, repeated measures of the participant's heart rate taken in the morning, afternoon, and evening. For *Categorical* repeated measures, neither the ordering nor the distance between the measures is meaningful. For example, repeated measures of breast density using three different instruments, Device A, B, and C.

Number of Measurements allows you to specify the number of times the repeated measure will be taken. For the blood pressure example, the *Number of Measurements* would be 6 because blood pressure was measured every 6 months. For numeric repeated measures, GLIMMPSE 2.0 auto-populates equidistant measurements. To change the distance between measures, type into the text boxes. For example, if blood pressure was measured every month for the first three months, then every other month for the next six months, the user would type 1, 2, 3, 5, 7, 9 into the text boxes.

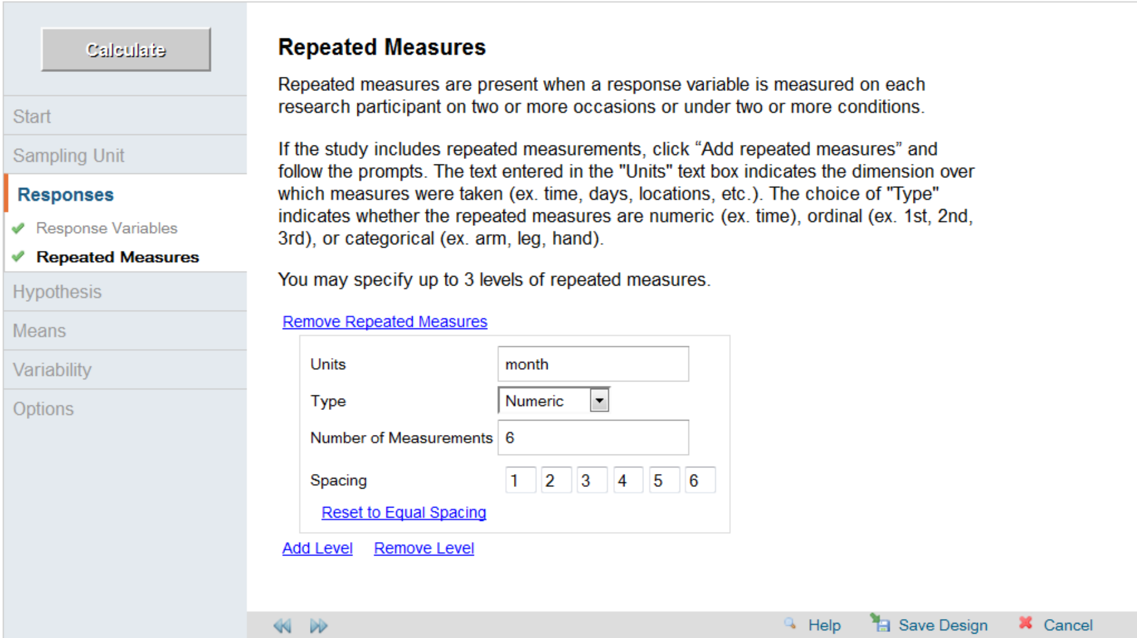

Figure 24: Repeated Measures

Nested repeated measures are added via the *Add Level* button. For example, consider a design in which a participant's blood pressure is measured every month for six months, and at each visit in three different positions (for example, standing, sitting, and supine). The design would include doubly repeated measures with one level for "month" and a second nested level for "position." The user may add up to three levels of repeated measures.

To add a sub-level, click the *Add level* button. Three more text boxes will appear:

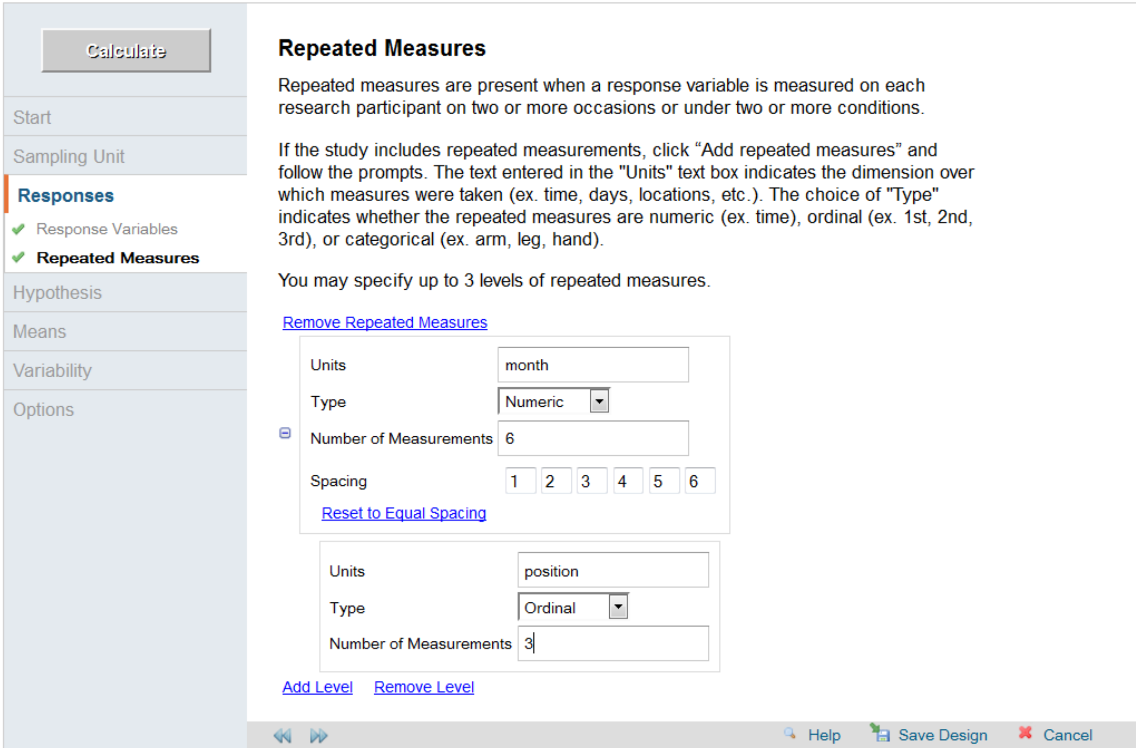

Figure 25: Repeated Measures: Add Level

When finished, click to proceed.

3.5. Hypotheses

3.5.1. Introduction

This screen provides an introduction to the *Hypothesis* section. After reading the information on the screen, click to proceed.

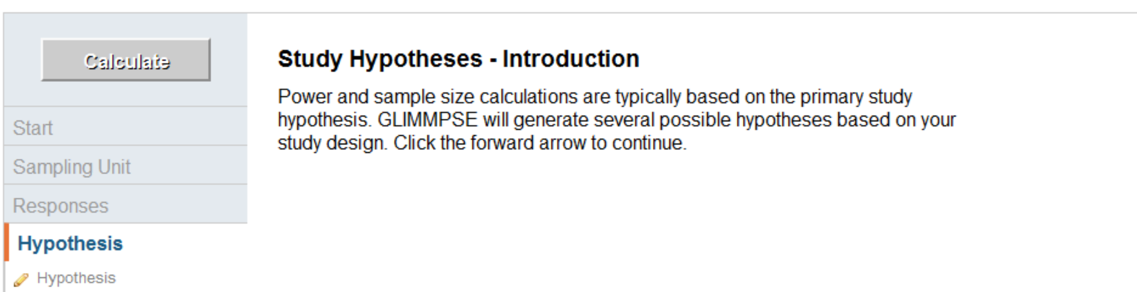

Figure 26: Hypothesis Introduction Screen

3.5.2. Hypotheses

The *Hypotheses* screen allows the user to select the primary hypothesis of interest. The user first selects the type

of hypothesis by clicking the appropriate radio button. Additional information will be requested depending on the type of hypothesis.

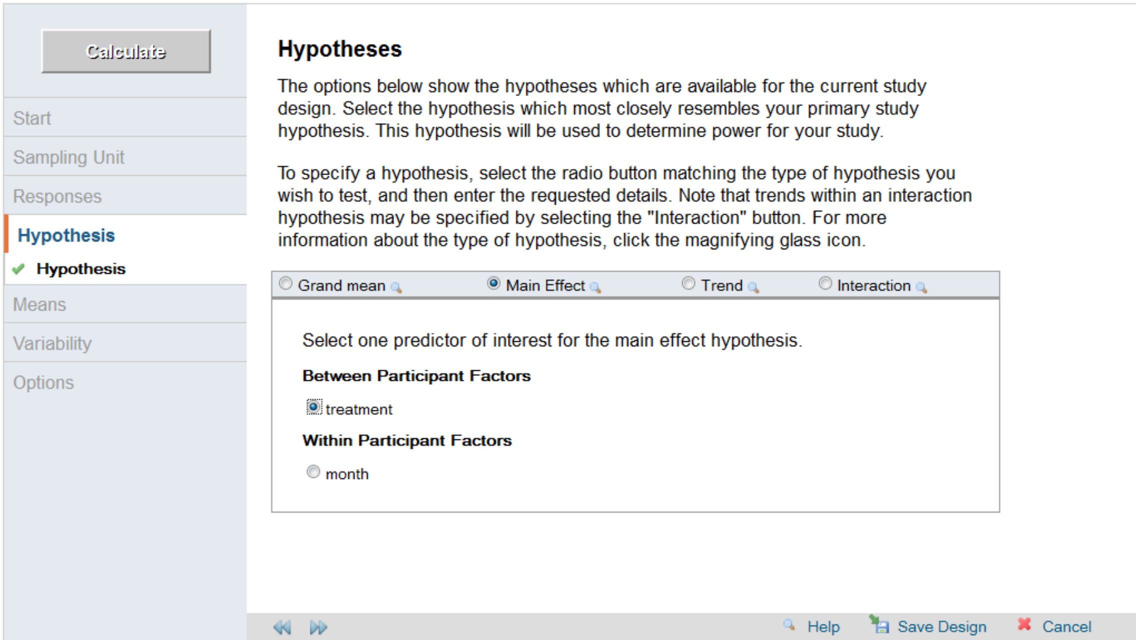

Figure 27: Hypotheses Screen

A *Grand Mean* hypothesis compares the overall mean response in a sample of participants against a known value. For example, an investigator may wish to determine if body mass index values for participants in a particular state differs from the United States national average. After selecting the *Grand mean* radio button, the user will be prompted to enter the known mean value for each response variable.

A *Main effect* hypothesis tests for the effect of a single predictor variable averaged across all other factors. For example, testing whether responses of participants in the treatment group differ on average from participant responses in a placebo group is a common main effect hypothesis. After selecting the *Main effect* radio button, the user will be prompted to select one predictor of interest by selecting the appropriate radio button.

A *Trend* hypothesis tests whether the effect of a single predictor follows a particular polynomial pattern, such as a linear or quadratic trend, across different levels of the predictor. After selecting the *Trend* radio button, the user will be prompted to select one predictor of interest. In addition, the user may select from six possible trends: no trend, change from baseline, linear trend, quadratic trend, cubic trend, or all polynomial trends.

An *Interaction* hypothesis tests whether the effect of one predictor changes depending on the value of one or more additional predictors. An interaction test also can be interpreted as a test of differences, as well as a test of parallel trajectories of response. For example, testing whether the effect of a cholesterol lowering medication on total serum cholesterol differs depending on the participant's gender is an example of an interaction hypothesis. After selecting the *Interaction* radio button, the user will be prompted to select one or more factors of interest by clicking the appropriate check boxes. In addition, the user may specify a trend for given factor by clicking the *Edit trend* button.

When finished, click to proceed.

3.6. Means

3.6.1. Introduction

This screen provides an introduction to the *Means* section. After reading the information on the screen, click to proceed.

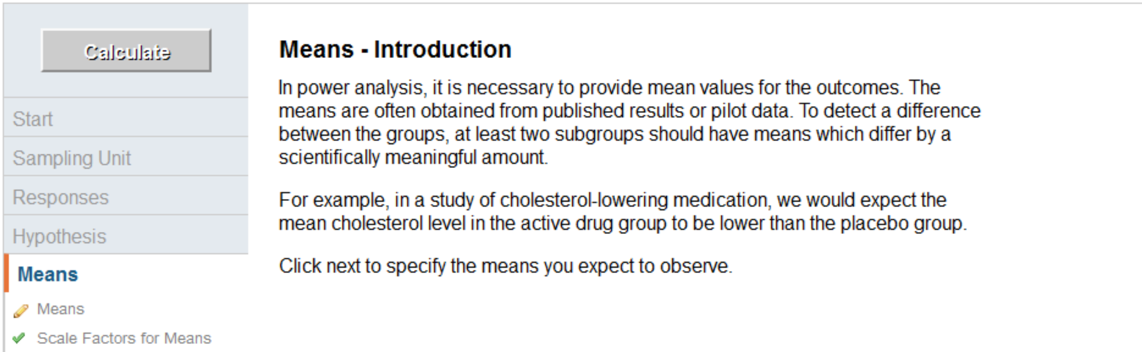

Figure 28: Means Introduction Screen

3.6.2. Means

The *Means* screen allows the user to enter the expected mean value for the experiment. Expected mean values are typically drawn from the literature or from pilot data. Differences between the entered means typically represent the smallest clinically relevant difference. The table should contain at least one value that is non-zero.

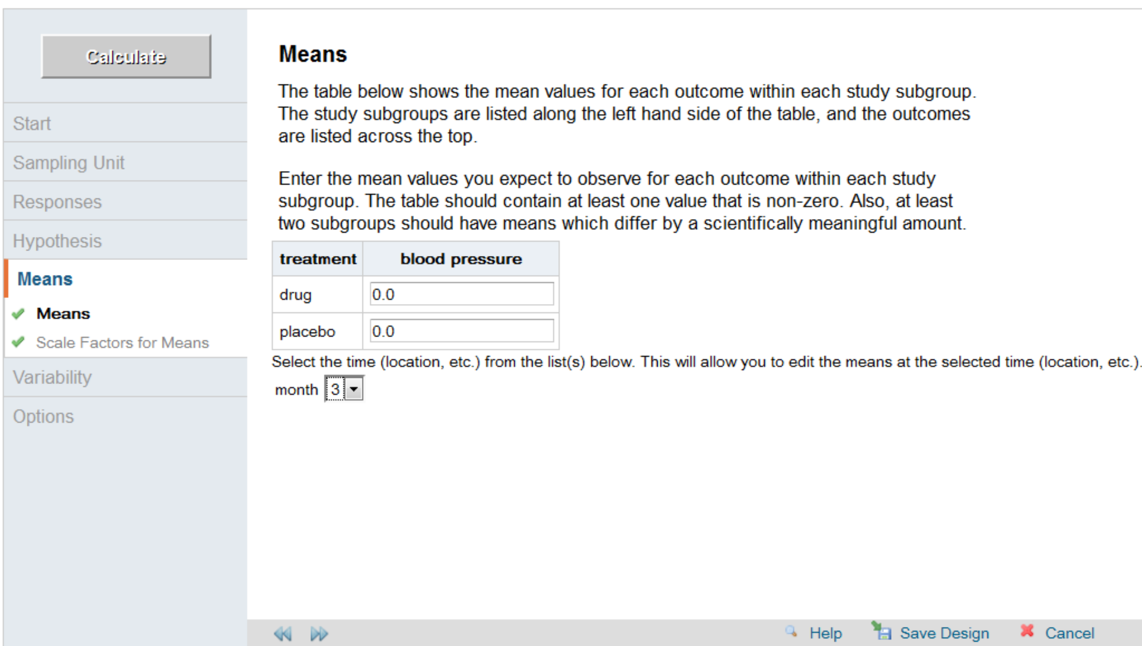

Figure 29: Means Screen

For designs with repeated measures, the user may enter means at each time (place, etc.). The drop down lists below the table of means allow the user to select a specific time (place, etc.) in order to edit the means.

When finished, click to proceed.

3.6.3. Flexible Means

The *Flexible Means* screen allows the user to compute power or sample size for the means as specified, the mean values divided by 2, and the mean values multiplied by 2. Leave the checkbox blank to compute power or sample size only for the means as entered on the *Means* screen.

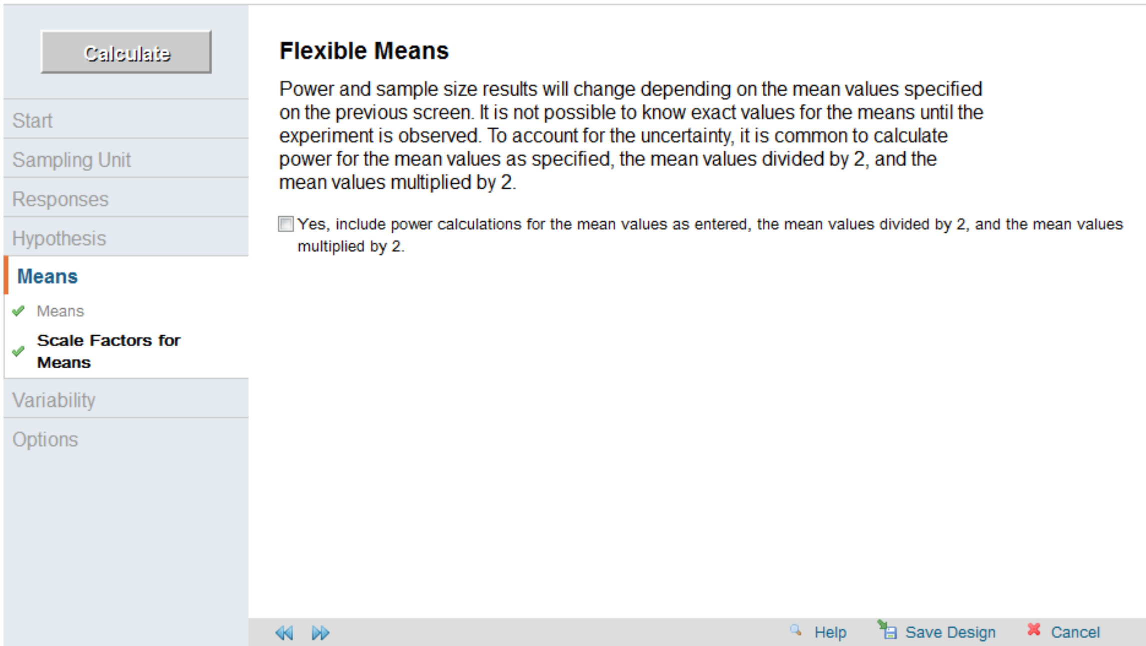

Figure 30: Scale Factors for Means Screen

When finished, click ⏩ to proceed.

3.7. Variability

3.7.1. Introduction

This screen provides an introduction to the *Variability* section. After reading the information on the screen, click ⏩ to proceed.

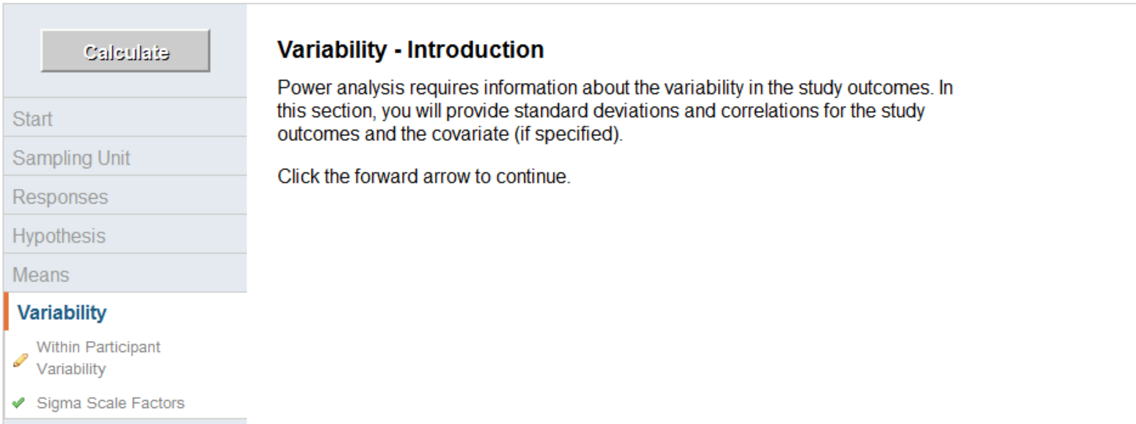

Figure 31: Variability Introduction Screen

3.7.2. Within Participant Variability

For a given participant, responses may vary across repeated measurements and for different response variables. The amount of variability can dramatically impact power and sample size. The *Within Participant Variability* screen allows the user to describe the variability he or she expects to observe for each within participant factor and response variable.

Separate tabs are presented for each "source" of correlation in the design. The *Responses* tab allows the user to specify the standard deviations of the response variables and any correlation between them. If repeated measures are present, a single tab will be presented for each level of repeated measures. Figure 32 shows an example design in which blood pressure is measured once a month for six months. GLIMMPSE will automatically combine the sources of correlation into a final covariance matrix.

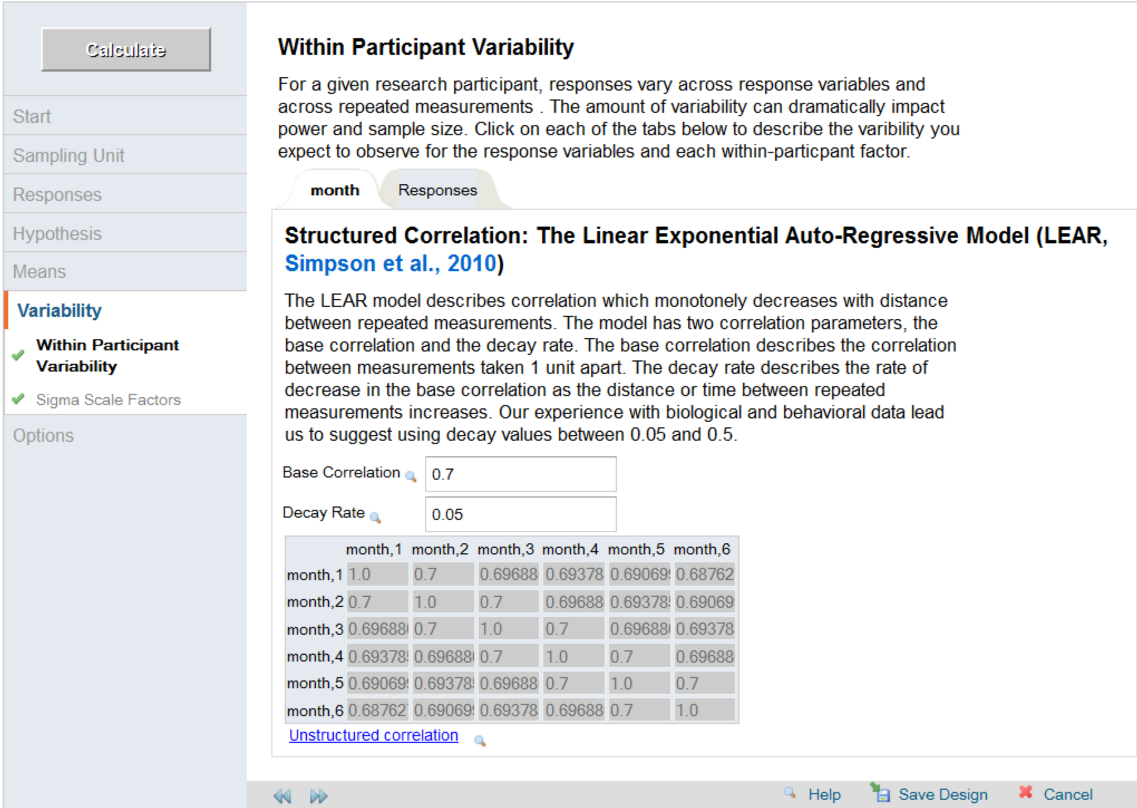

Figure 32: Within Participant Variability

When finished, click to proceed.

3.7.3. Sigma Scale Factors

While GLIMMPSE requests standard deviations, it actually computes variances when it conducts the power or sample size calculations. There may be considerable uncertainty about what standard deviation or variance value to use. To account for this uncertainty, it is common to calculate power or sample size using alternative values for variability. The *Flexible Variability* screen allows you to compute power for half the variance, the variance as specified, and twice the variance. This generates scale factors of 0.5, 1, and 2 for the covariance matrix. If you wish to have a range of variances, check the checkbox.

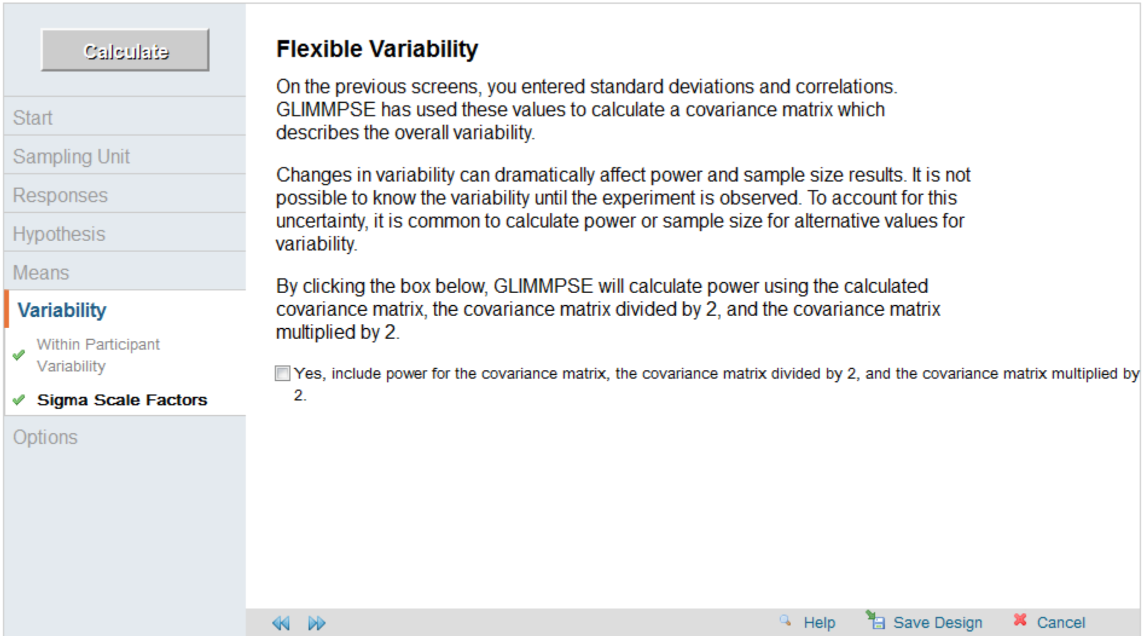

Figure 33: Flexible Variability

When finished, click to proceed.

3.8. Options

This screen provides an introduction to the Options section. After reading the screen, click to proceed.

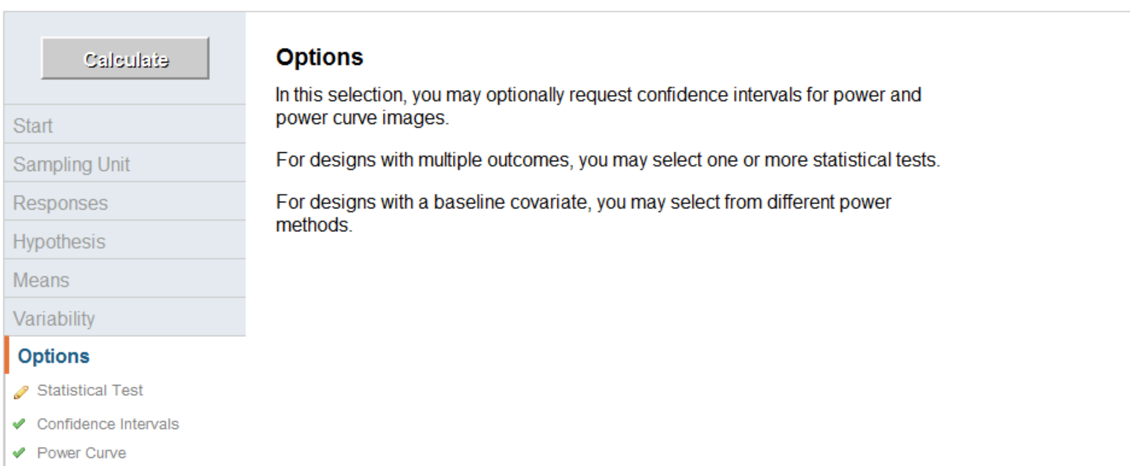

Figure 34: Options

3.8.1. Statistical Tests

The *Statistical Tests* screen allows the user to select one or more statistical tests for the power or sample size calculations. A tutorial providing guidelines for selecting a test is available from the GLIMMPSE Tutorials page

at http://samplesizeshop.org/education/tutorials. Select the statistical test(s) you wish to use by clicking one or more check boxes.

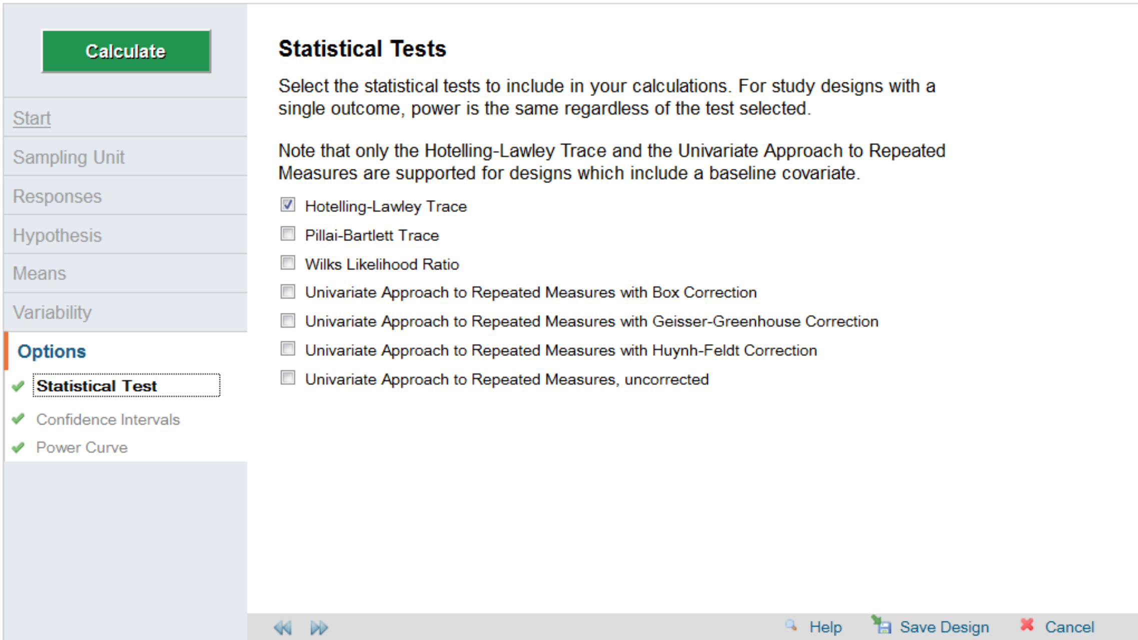

Figure 35: Statistical Tests

When finished, click to proceed.

3.8.2. Power Calculation Method

For designs with a baseline covariate, two different methods are available to calculate power: quantile and unconditional power. For theoretical details, please see Glueck and Muller (2003). Select the power methods by clicking the check boxes. If quantile power is selected, the user must also specify one or more quantile values. For example, median power would be obtained by selecting *Quantile* power and entering “0.5” in the quantile list box.

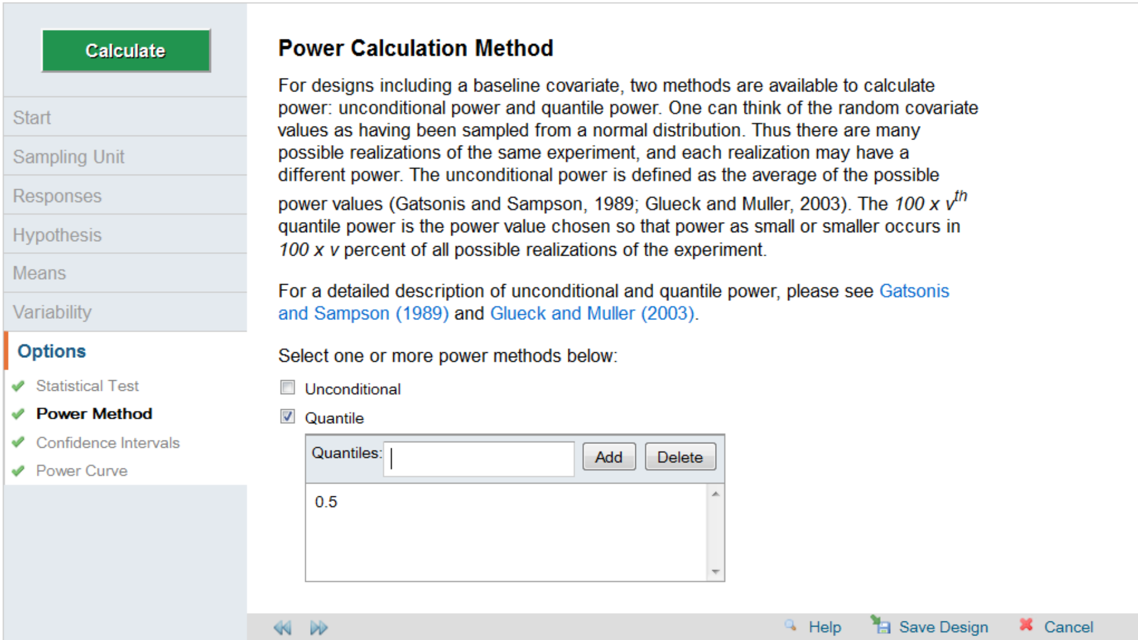

Figure 36: Statistical Tests

When finished, click to proceed.

3.8.3. *Confidence Intervals*

Power analysis involves some uncertainty in the choices for means and variability. Therefore, the *Confidence Intervals* screen allows the user to request confidence intervals on the power results. To include confidence intervals, uncheck the checkbox. The information on the confidence interval screen describes the data set (or publication) from which the choices for means and variances were obtained. For example, if a scientist were calculating power based on the means and variances obtained from pilot data, the scientist would enter information describing the pilot data set. The following information is required:

The *Assumptions* section allows the user to indicate if he or she is uncertain about the variance, but reasonably certain of the mean values, or uncertain of both the means and variance.

The *Upper and lower tail probabilities* define the width of the confidence interval. For example, a centered 95% confidence interval would have both upper and lower tail probabilities of 0.025.

The *Total sample size* value indicates the number of independent sampling units in the pilot data set (or publication).

The *Rank of the design matrix* describes a property of the predictor matrix used in the pilot data set. Please see Muller and Stewart (2006) for details about matrix rank.

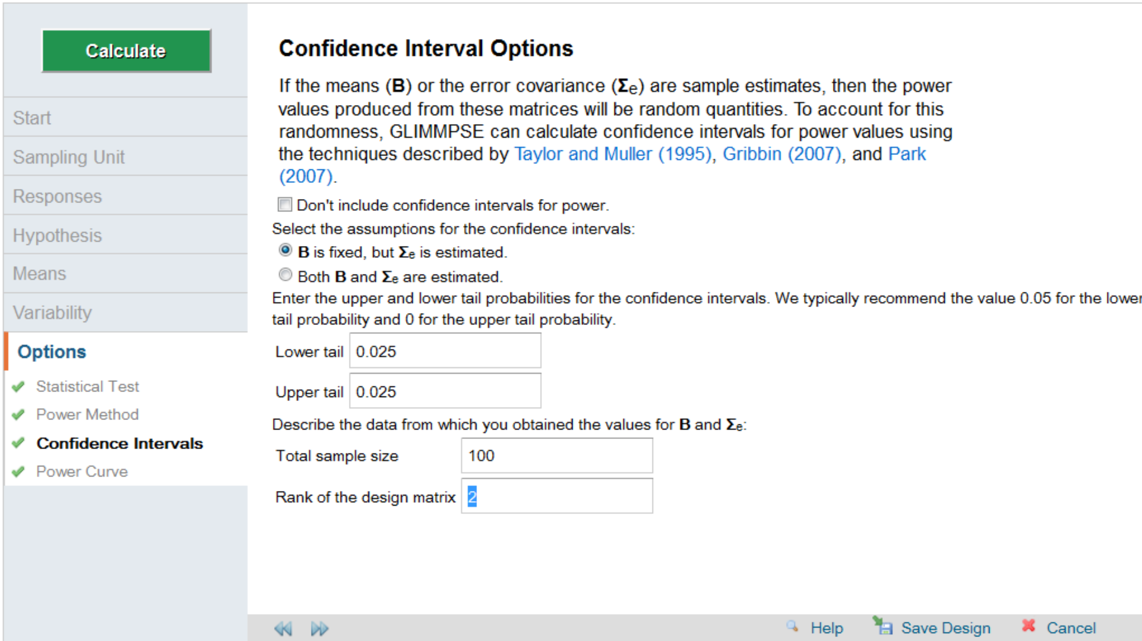

Figure 37: Confidence Intervals

When finished, Click to proceed.

3.8.4. Power Curve Options

The *Power Curve Options* screen allows the user to create a power curve. A power curve describes the change in power (Y axis of the power curve) relative to the total sample size, regression coefficient scale factor, or the variability scale factor (all options for the X axis of the power curve).

To create a power curve, the user must 1) uncheck the check box, 2) select the value to appear on the horizontal axis, and 3) add one or more data series.

Depending on the study design, the user may request a large number of power or sample size values in a single request. A data series is defined by selecting a subset of the power or sample size values. The user creates a data series by selecting values for several study design variables and clicking the Add button. A data series will be displayed as a single line on the power curve plot.

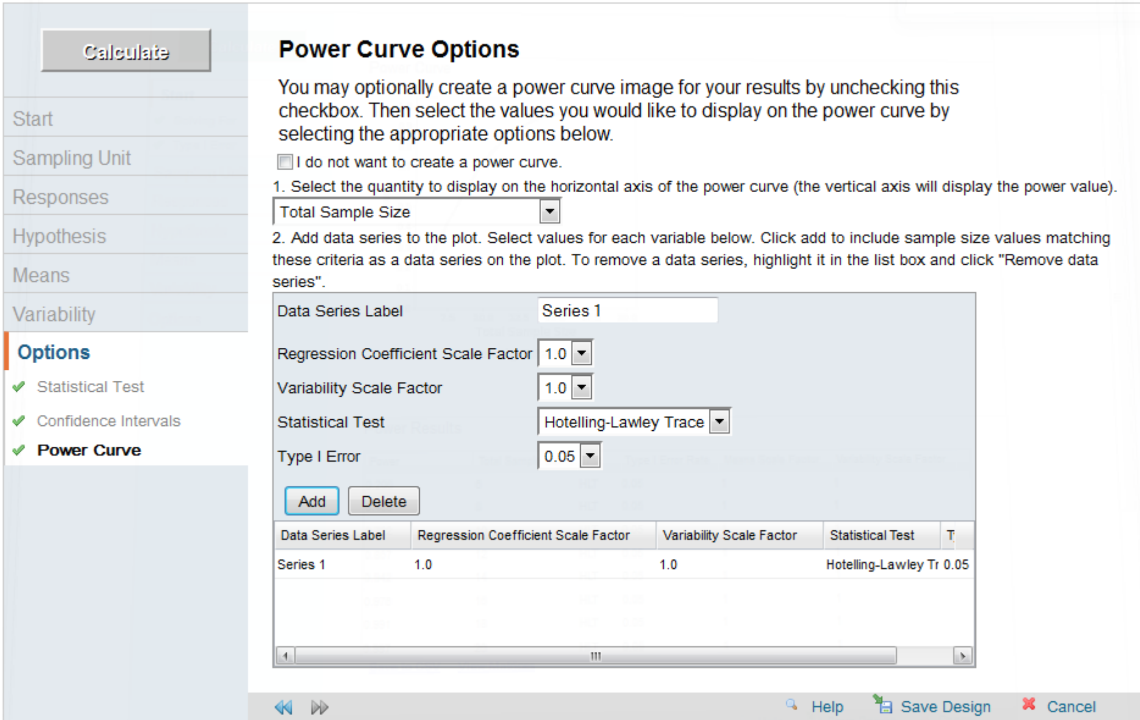

Figure 38: Power Curve

When finished, click to proceed to the results screen. Note that the *Power Curve Options* screen is the final screen in the GLIMMPSE wizard. If the study design is not complete, the button will be disabled.

3.9. Calculate

When sufficient information for your power or sample size calculation has been entered, the *Calculate* button will be highlighted green. Click Calculate to receive the results of your power analysis. Example results are shown in Figure 39. For detailed information regarding the Power Results table, refer to Table 1.

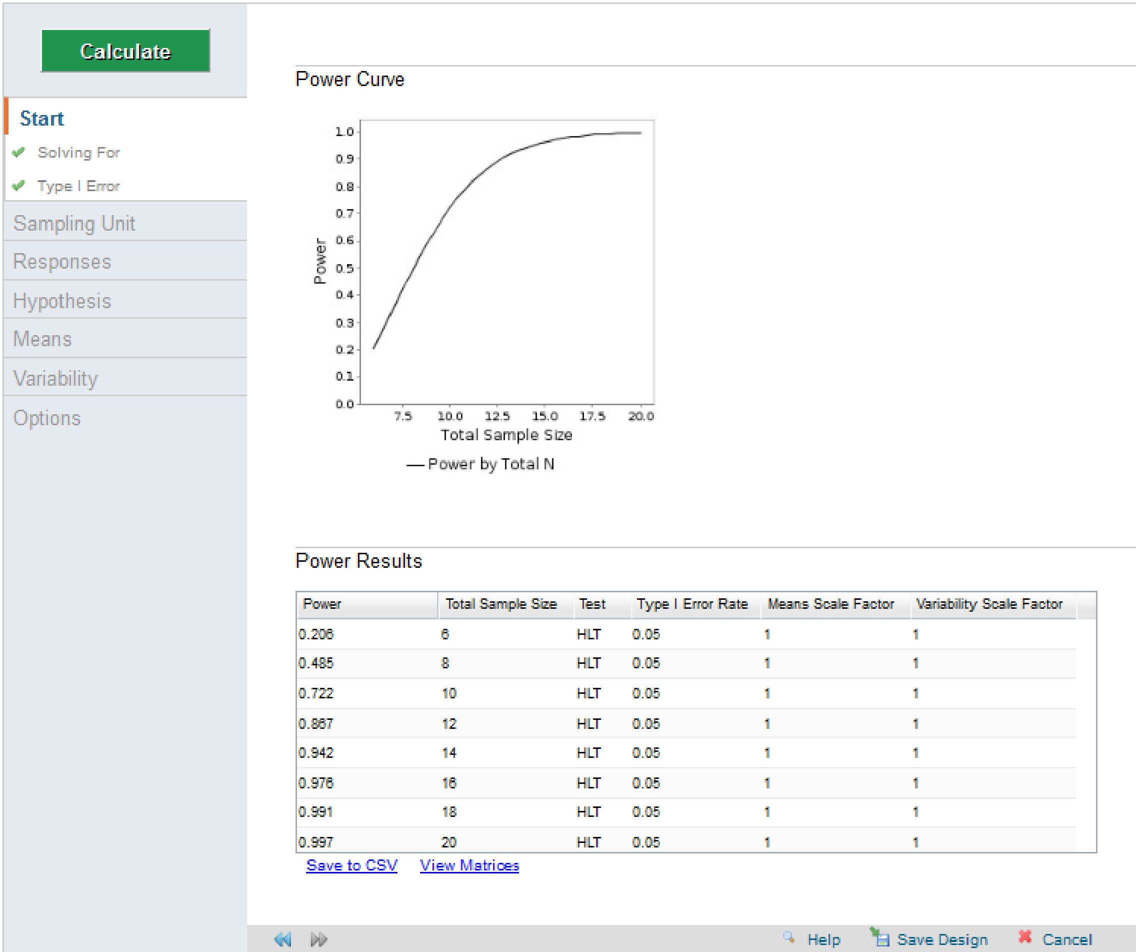

Figure 39: Results

4. *Matrix Mode* Screen-by-Screen Tour

Matrix Mode allows direct input of all matrices required for a power calculation. In *Matrix Mode* users receive less guidance than in *Guided Mode*, and are assumed to possess in-depth statistical training.

4.1. Start

4.1.1. Introduction

The *Introduction* screen briefly describes the required matrix inputs for the power or sample size calculation.

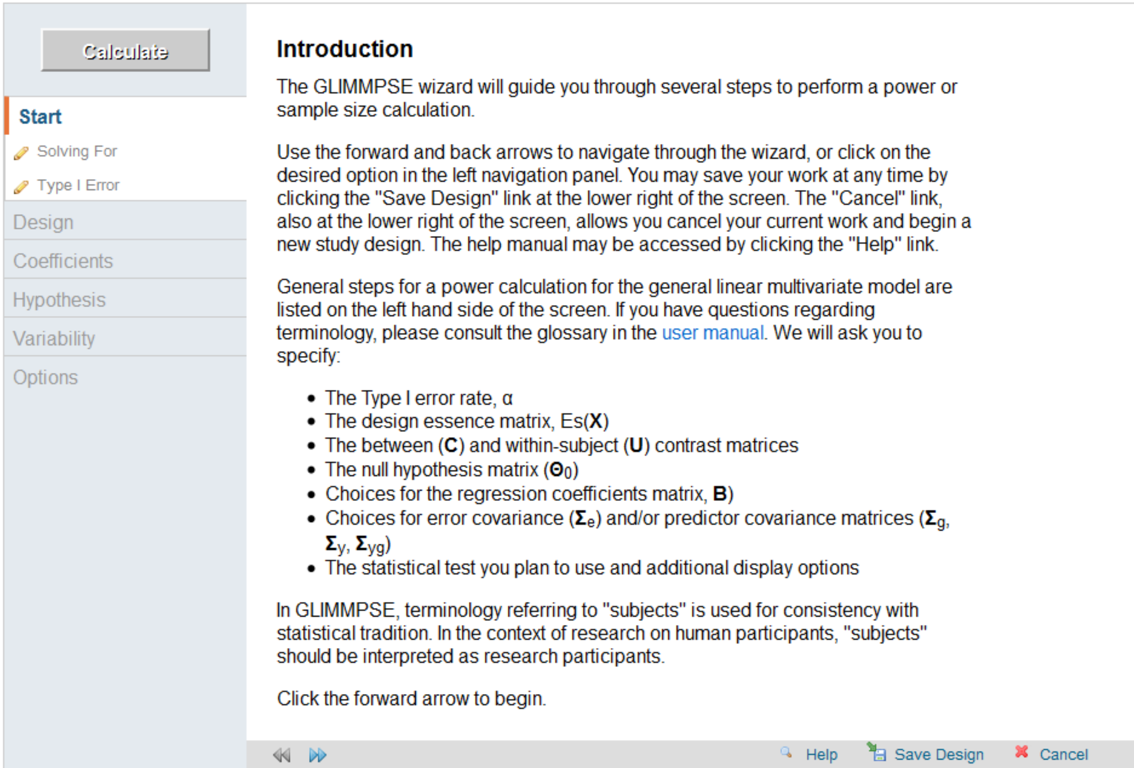

Figure 40: Introduction Screen for *Matrix Mode*

Click to begin entering the details of the analysis.

4.1.2. Solving For?

The *Solving For?* screen allows the user to select either a power or sample size calculation.

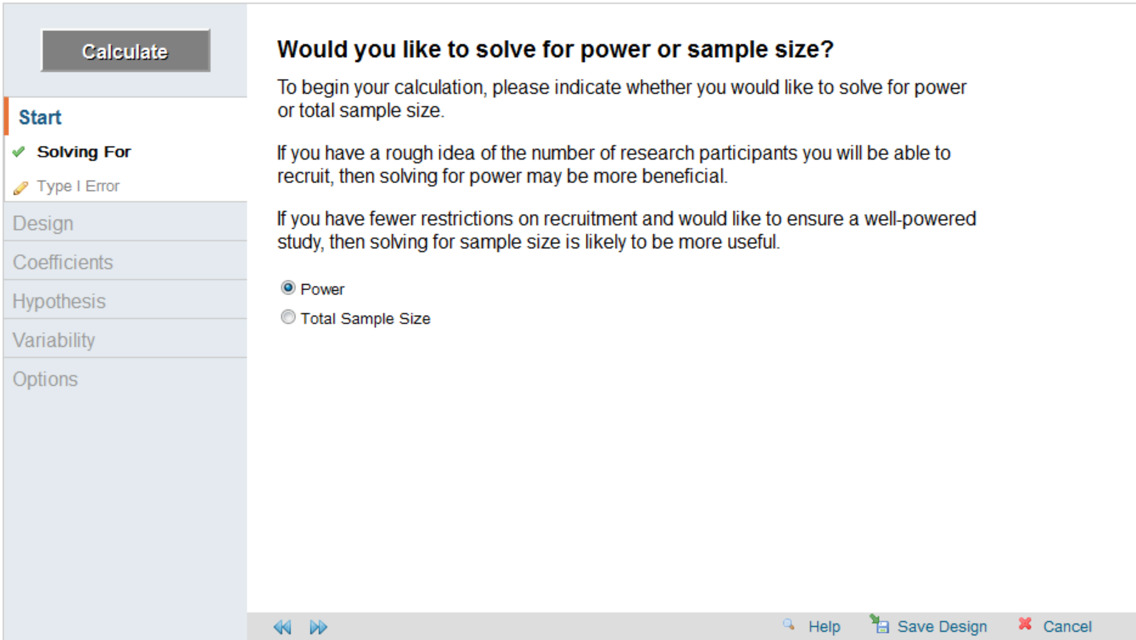

Figure 41: Solving For? Screen

When *Power* is selected, the inputs will be used for a power analysis. The power analysis will produce a value(s) between 0 and 1, representing the probability the study will answer the question of interest. When *Total Sample Size* is selected, the inputs will be used to calculate the number of individual sampling units (also called participants, if referring specifically to people) needed for the study to achieve the desired power.

If the number of participants is not set, we recommend solving for sample size in order to obtain the appropriate sample size for achieving the goals of your study. However, if sample size is set due to budgetary or other restrictions, a power calculation will indicate the probability that the study will provide a definitive answer to the question of interest.

On the screen, select *Power* or *Total Sample Size* by selecting the appropriate radio button.

Upon completing the selection, click to proceed.

4.1.3. Desired Power (if solving for Total Sample Size)

When solving for sample size, the user must enter the desired power for the study. Enter the target values as decimals (i.e. 0.95) in the Power Values box and click Add to add the value to the list.

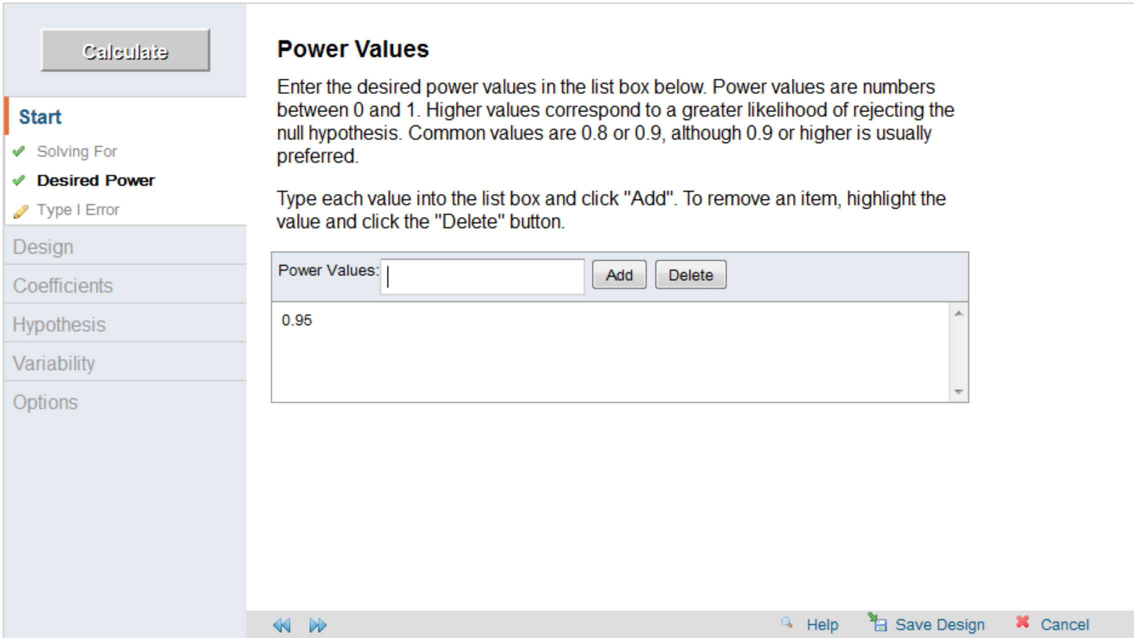

Figure 42: Desired Power Screen

When finished, click to proceed.

4.1.4. Type I Error

Enter the target values for Type I Error as decimals (i.e. 0.05) in the Type I Error Values box. The user may specify up to five Type I Error values.

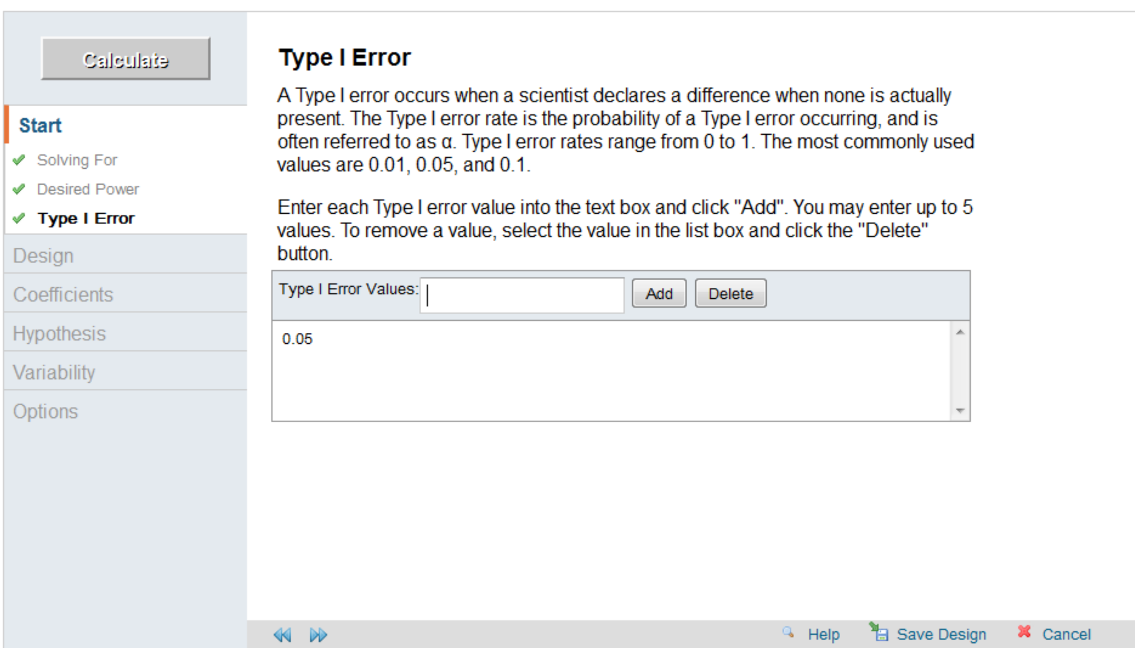

Figure 43: Type I Error

When finished, click ⏩ to proceed.

4.2. Design

4.2.1. Design Essence

In the *Design* section, the user will define the composition of the study by specifying the number of groups, how subjects are divided into groups, the size of each group, and whether the design will include a Gaussian covariate.

The Design Essence Matrix

In the general linear multivariate model with fixed predictors, $\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{B} + \boldsymbol{E}$, the $\boldsymbol{X}$ matrix represents the study design. The same is true for $\boldsymbol{F}$ in the general linear multivariate model with fixed predictors and a Gaussian predictor (Glueck and Muller 2003). For simplicity, we will only discuss $\boldsymbol{X}$ (since the instructions do not change for $\boldsymbol{F}$). In data analysis, the $\boldsymbol{X}$ matrix would contain a single row for each subject. Since power analysis does not include actual data, the design "essence" matrix (Muller and Stewart 2006) is a version of the $\boldsymbol{X}$ matrix that contains a single row for each unique combination of predictors in the study design. Note that the essence matrix specifies only the fixed, or categorical, predictors in the study design.

For example, consider a 2-factor ANOVA design with 2 levels per factor, 3 subjects per group, and a cell means coding. In data analysis, the design matrix and corresponding essence matrix would be:

$$\boldsymbol{X} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \implies \mathrm{Es}\left(\boldsymbol{X}\right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

GLIMMPSE requires that the design coding is full rank. Unequal group sizes may be coded by replicating a row to reflect the relative sizes of the groups.

After entering the desired dimensions for the matrix in the row and column dimension text boxes, click anywhere on the screen for the matrix to be resized. Type in the matrix text boxes to enter the matrix information.

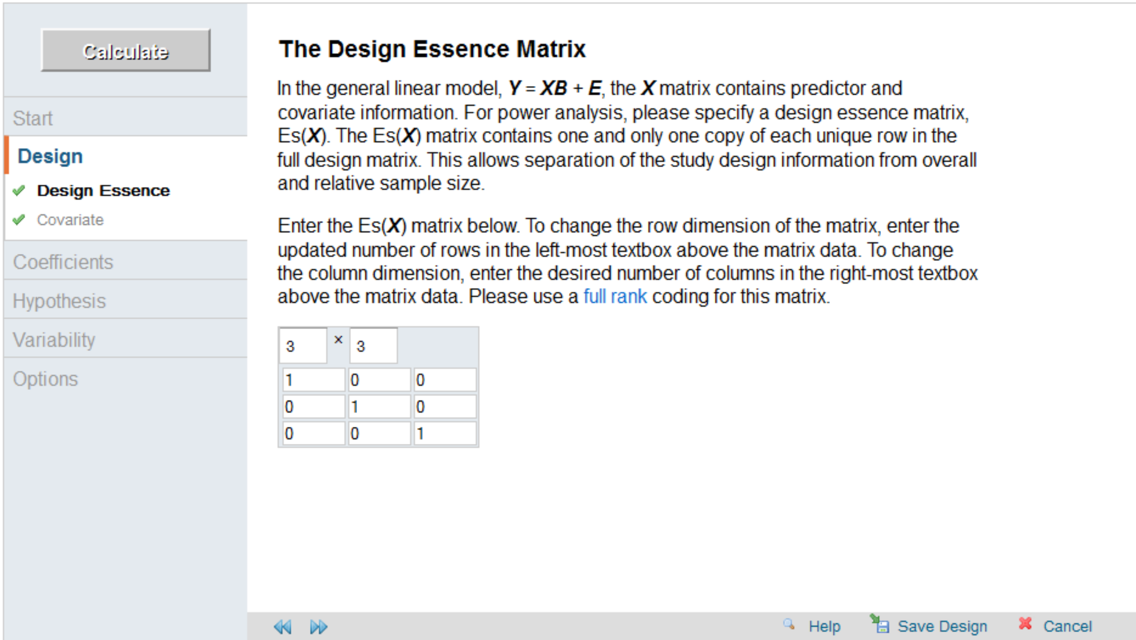

Figure 44: Type I Error

When finished, click to proceed.

4.2.2. Covariate

Currently, GLIMMPSE only performs power calculations for hypotheses about fixed predictor variables. However, a single, continuous, normally distributed predictor variable may be included in the analysis.

To include such a predictor, click the checkbox next to *Control for a single, normally distributed Gaussian predictor* at the bottom of the screen.

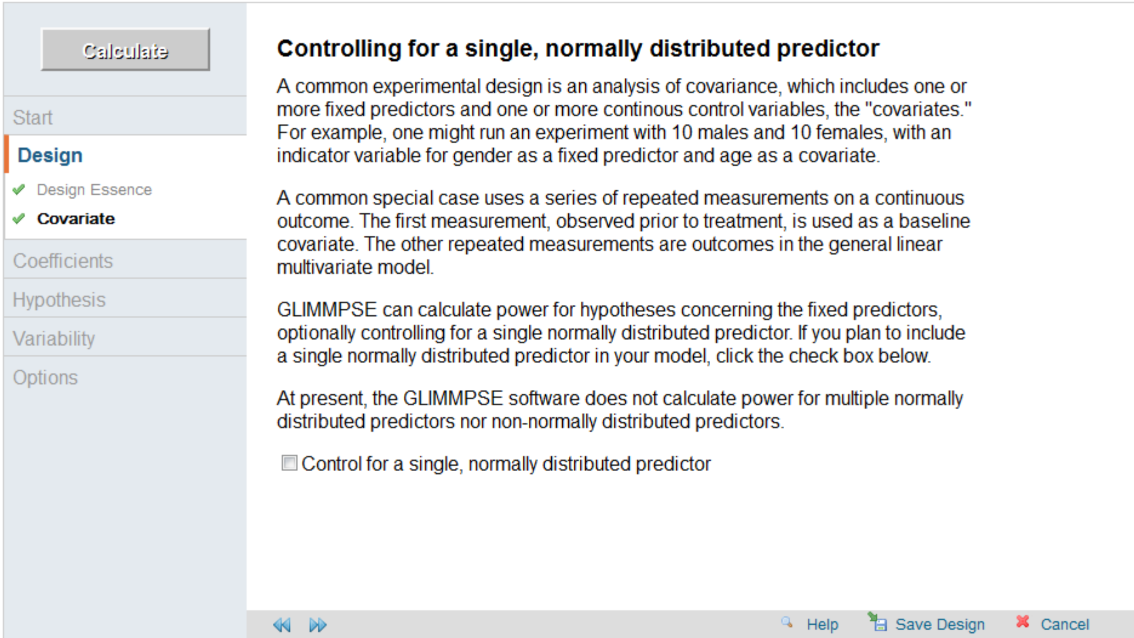

Figure 45: Gaussian Predictor Screen

When finished, click to proceed.

4.2.3. Smallest Group Size

When solving for power, the user specifies the total sample size for the design by the relative number of repeated rows in the design essence matrix, and the smallest group size. On the *Smallest Group Size* screen, the user may enter one or more values describing the number of participants in the smallest group.

To enter one or more per group sample size, type the sample size in the *Per Group Sample Size* box. After each entry, click Add, press Enter on your keyboard, or click anywhere on the screen. To delete a value, select the unwanted value and click Delete to remove the value from the list.

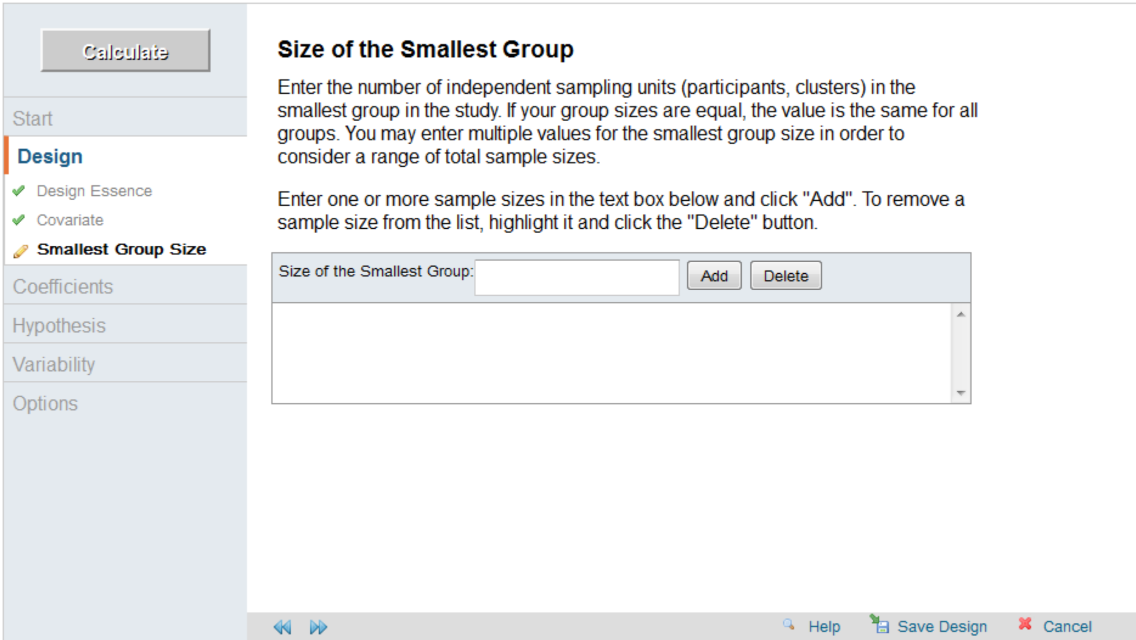

Figure 46: Smallest Group Size Screen

When finished, click to proceed.

4.3. Coefficients

4.3.1. Beta Coefficients: B Matrix

This section requires the user to enter choices for values for the hypothesis test, $\boldsymbol{\Theta} = \boldsymbol{CBU}$.

In the general linear multivariate model with fixed predictors, $\boldsymbol{Y} = \boldsymbol{XB} + \boldsymbol{E}$, the $\boldsymbol{B}$ matrix represents the proposed relationship between the predictor variables, $\boldsymbol{X}$, and the outcome variables, $\boldsymbol{Y}$. The same is true for $\boldsymbol{B}_F$ in the General Linear Multivariate Model with Fixed Predictors and a Gaussian Predictor. For simplicity, we will only discuss $\boldsymbol{B}$ (since the instructions do not change for $\boldsymbol{B}_F$). To calculate power, enter values for the regression coefficients for each unique combination of predictors in the study design. The row dimension of $\boldsymbol{B}$ is determined by the number of columns in the essence matrix. Change the column dimension of $\boldsymbol{B}$ to match the intended number of outcomes in the study, or the columns of $\boldsymbol{Y}$ in the general linear multivariate model with fixed predictors regression equation.

For example, an investigator may want to compare vitamin D and calcium levels of children who live in three different regions: urban, suburban, and rural. The $\boldsymbol{B}$ matrix would have three pre-specified rows, one for each region, and two columns, one for vitamin D and one for calcium. To calculate power, the investigator must enter the expected mean vitamin D (column 1) and calcium (column 2) levels of the children in the rural (row 1), suburban (row 2), and urban (row 3) regions. The investigator may choose $\boldsymbol{B} = \begin{bmatrix} 120 & 4 \\ 60 & 8 \\ 45 & 10 \end{bmatrix}$ as shown below:

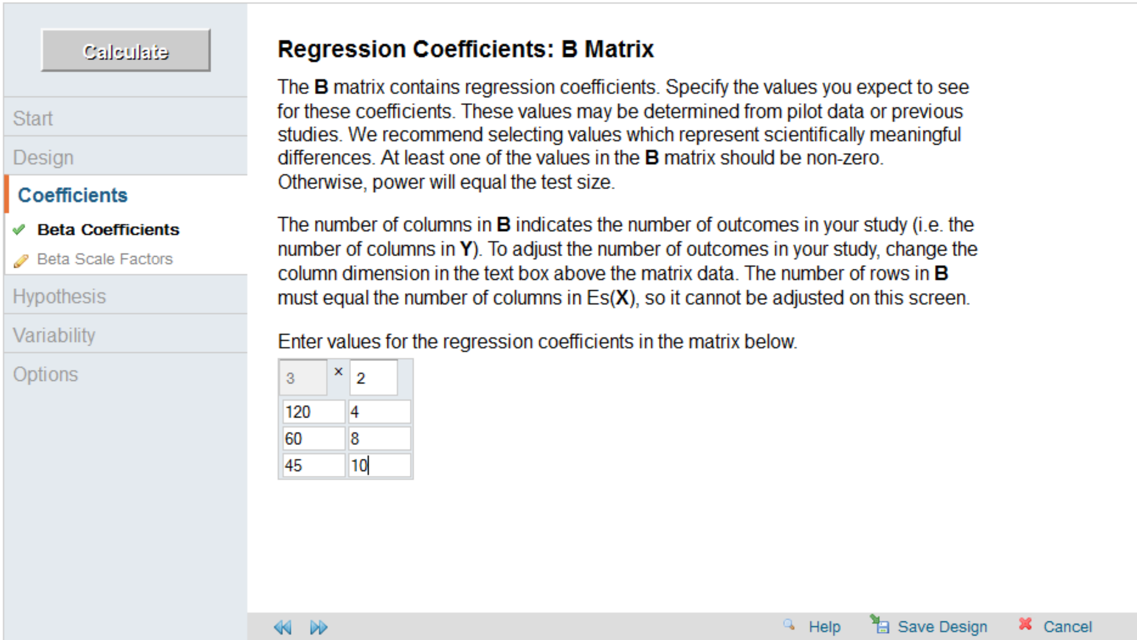

Figure 47: Beta Matrix

Enter the number of columns, or the number of outcomes in the study, in the column text box (right) in the matrix. Press Enter on your keyboard or click anywhere on the screen to resize the blank matrix. Enter proposed values of the $\boldsymbol{B}$ coefficients in their corresponding text boxes in the matrix.

When finished, click to proceed.

4.3.2. Beta Scale Factors

GLIMMPSE allows users to specify scale factors for the $\boldsymbol{B}$ matrix in order to generate power or sample size values for different coefficient values. Since power is based on proposed regression coefficients, it is common to calculate power for the proposed value, as well as alternative values such as half and twice the proposed value.

One or more scale factors for the $\boldsymbol{B}$ matrix may be specified for inclusion in the power calculation. For example, to calculate power for regression coefficients that are half the values in your $\boldsymbol{B}$ matrix, enter 0.5. To use the exact $\boldsymbol{B}$ matrix specified, enter 1. After each entry, click Add, press Enter, or click anywhere on the screen. To delete a value, select the unwanted value and click Delete to remove the value from the list.

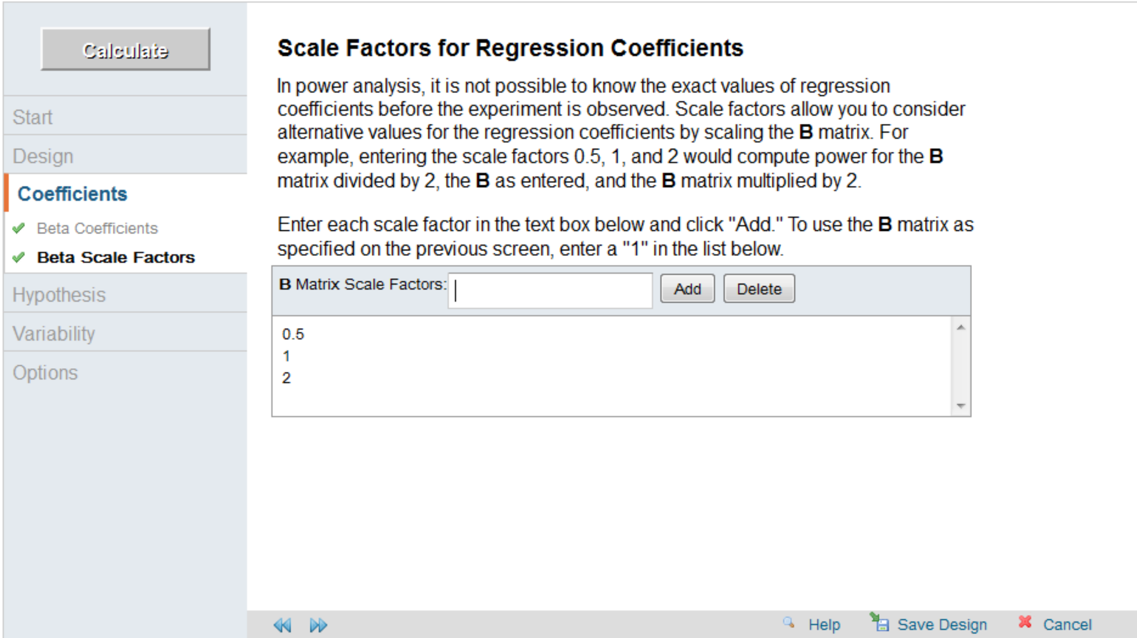

Figure 48: Beta Matrix

When finished entering your values, click to proceed.

4.4. Hypothesis

In this section, the user defines the contrast matrices in the study. The contrast matrices, $\boldsymbol{C}$ and $\boldsymbol{U}$, consist of the hypotheses to be tested. They are used to calculate the expected hypothesis matrix, $\boldsymbol{\Theta} = \boldsymbol{CBU}$.

4.4.1. Between-Participant Contrast

The $\boldsymbol{C}$ matrix consists of the between-participant contrasts. The between-participant contrasts test hypotheses between independent sampling units. The number of rows in the $\boldsymbol{C}$ matrix represent the degrees of freedom for the hypothesis test. For example, suppose an investigator wants to compare the average final exam test scores of students in class A and class B. The contrast matrix would be $\boldsymbol{C} = \begin{bmatrix} 1 & -1 \end{bmatrix}$. When multiplied by $\boldsymbol{B}$, this becomes the difference in the proposed average test scores between class A and class B.

Enter the number of rows/number of contrasts in the study, in the row text box (left) under $\boldsymbol{C}$ Matrix. Press Enter on your keyboard or click anywhere on the screen to resize the blank matrix. Fill in the contrasts you wish to test in the matrix.

The number of rows in the $\boldsymbol{C}$ matrix cannot exceed the number of rows in the essence matrix minus 1. In addition, the $\boldsymbol{C}$ matrix must conform to the $\boldsymbol{B}$ matrix, so the number of columns cannot be adjusted on this screen.

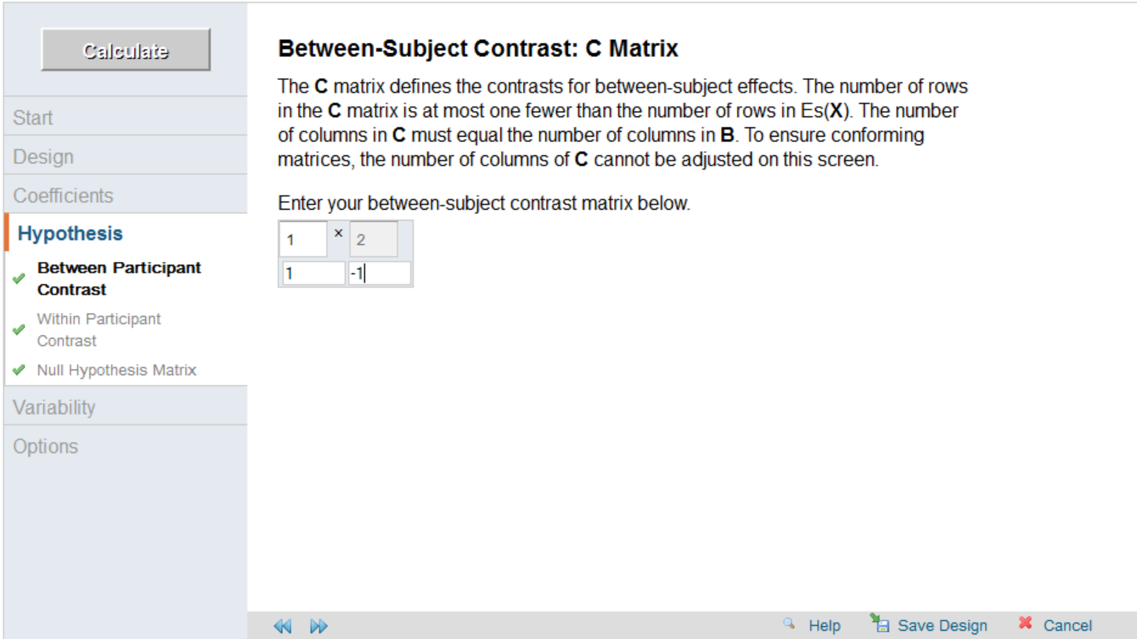

Figure 49: Between-participant Contrast Matrix

When you have completed the matrix, click  to proceed.

4.4.2. Within-Participant Contrast

The $\boldsymbol{U}$ matrix consists of the within-participants contrasts. The within-participants contrasts are the hypotheses that compare measurements on the same independent sampling unit.

The $\boldsymbol{U}$ matrix is most useful for multivariate designs and repeated measures. For example, suppose an investigator wants to examine whether student test scores improve from their midterm exams to their final exams. The investigator would have two measurements per student, one for the midterm and one for the final. The within-participant contrast matrix would be $\boldsymbol{U} = \begin{bmatrix} 1 & -1 \end{bmatrix}$. The matrix contrasts two different test scores, the midterm and the final, for the same student.

Enter the number of columns, or the number of within-subject contrasts, in the study, in the column text box (right). Press Enter on your keyboard or click anywhere on the screen to resize the blank matrix. Fill in the contrasts in the matrix.

The $\boldsymbol{U}$ matrix must conform to the $\boldsymbol{B}$ matrix, so the number of rows cannot be adjusted on this screen.

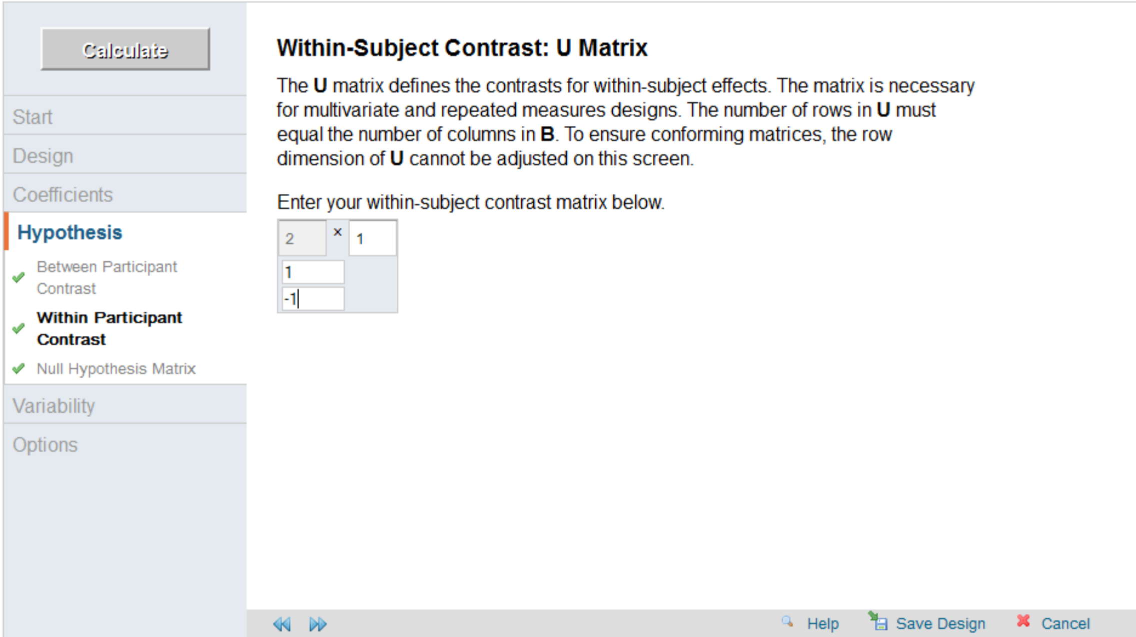

Figure 50: Within-participant Contrast Matrix

When finished, click ⏩ to proceed.

4.4.3. Null Hypothesis

The null hypothesis matrix, $\mathbf{\Theta}_0$, represents the test values the user expects to observe when the null hypothesis is true. When performing a power analysis, the values for the hypothesis tests are calculated as $\mathbf{\Theta} = \boldsymbol{CBU}$, and then compared against $\mathbf{\Theta}_0$. Commonly, $\mathbf{\Theta}_0$ is a matrix of zeroes.

For example, suppose an investigator wants to compare resting metabolic rate between subjects with HIV lipoatrophy, subjects with HIV only, and healthy controls. The null hypothesis of no difference between the three groups is $\mathbf{\Theta}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, appearing as follows:

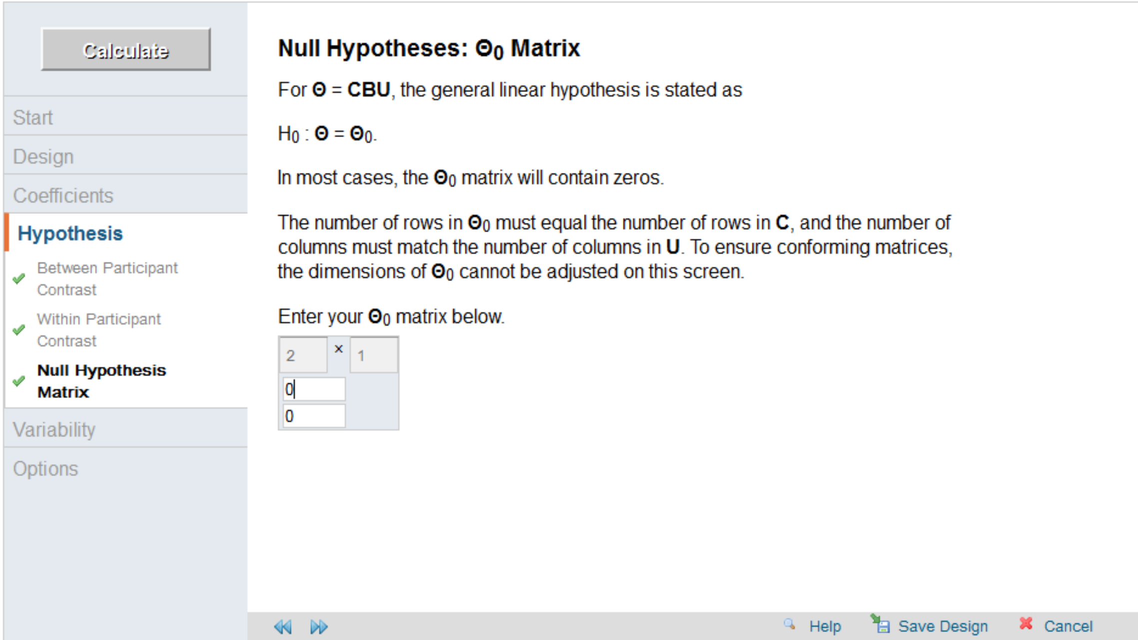

Figure 51: Null Hypothesis Matrix

Sometimes, however, the null hypothesis is based on previous studies or clinical experience. For example, suppose an investigator wants to compare foal birth weight between dams who are given feed formula A, feed formula B, and standard feed. In order to be cost effective, the new feed formulas must improve foal birth weight by more than 7 kg. The null hypothesis, then is $\mathbf{\Theta}_0 = \begin{bmatrix} 7 \\ 7 \end{bmatrix}$, appearing as shown below.

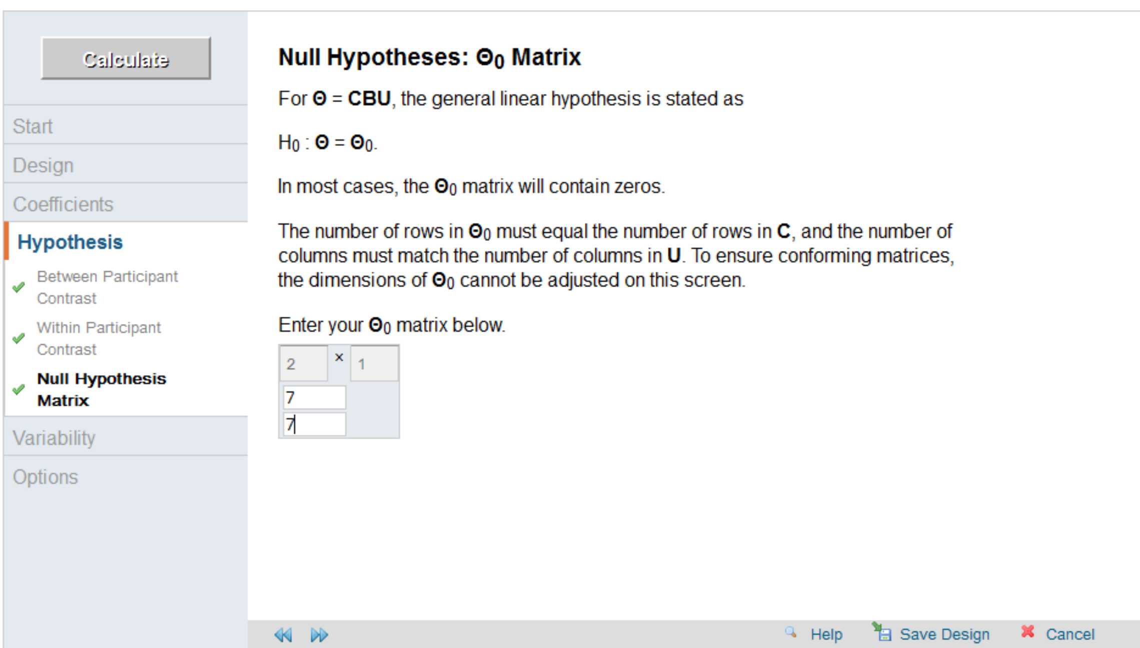

Figure 52: Non-zero Null Hypothesis Matrix

Θ_0 has the same number of rows as C, and the same number of columns as U. Therefore, its size cannot be adjusted on this screen. The user need only enter the matrix cell values.

When finished specifying the Θ_0 matrix, click to proceed.

4.5. Variability

Variability describes how much measurements differ from each other. In this section, the user defines the covariance of errors and covariance related to the Gaussian covariate.

4.5.1. Error Covariance

For each independent sampling unit, Σ_e is the covariance of the random errors conditional on the values of the fixed predictors. The *Error Covariance* screen allows the user to define Σ_e by directly entering the covariance matrix values. To ensure conformance with the B and U matrices, the dimensions of the Σ_e matrix cannot be modifed on this screen.

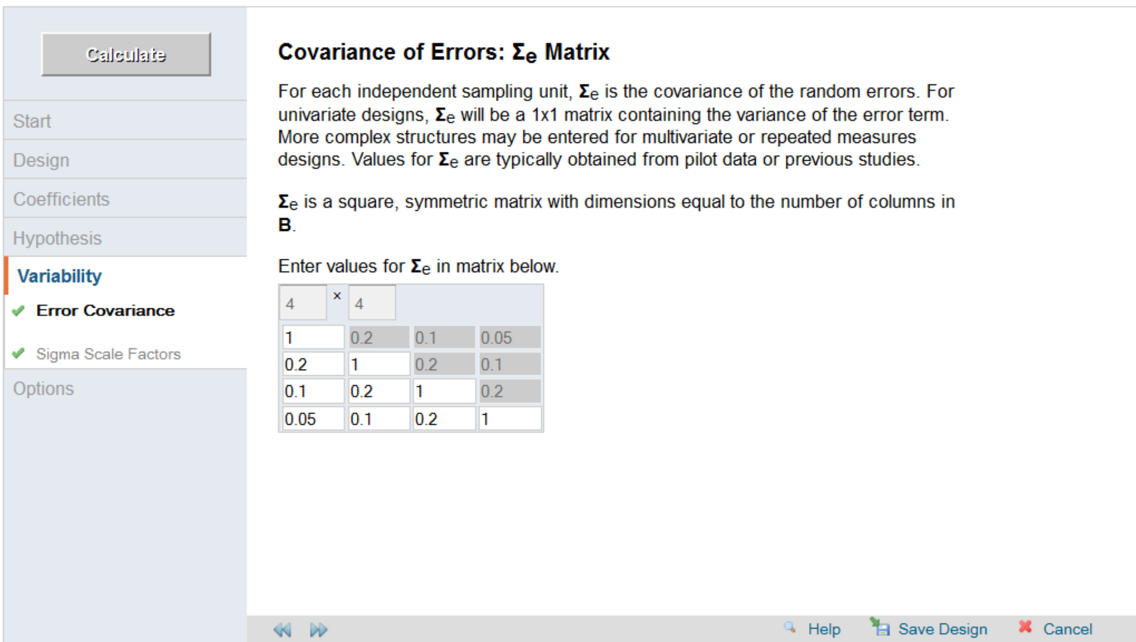

Figure 53: Error Covariance

When finsihed, click to proceed.

4.5.2. Outcomes Covariance

For designs with a Gaussian covariate, the user must specify the covariance of the outcomes, Σ_Y. For each independent sampling unit, Σ_Y is the covariance of the outcomes conditional on the fixed predictors. One can think of Σ_Y as the error covariance for each independent sampling unit in a model containing only the fixed predictors and excluding the Gaussian covariate. The *Outcomes Covariance* screen allows the user to define Σ_Y by directly entering the covariance matrix values. To ensure conformance with the B and U matrices, the dimensions of the Σ_Y matrix cannot be modifed on this screen.

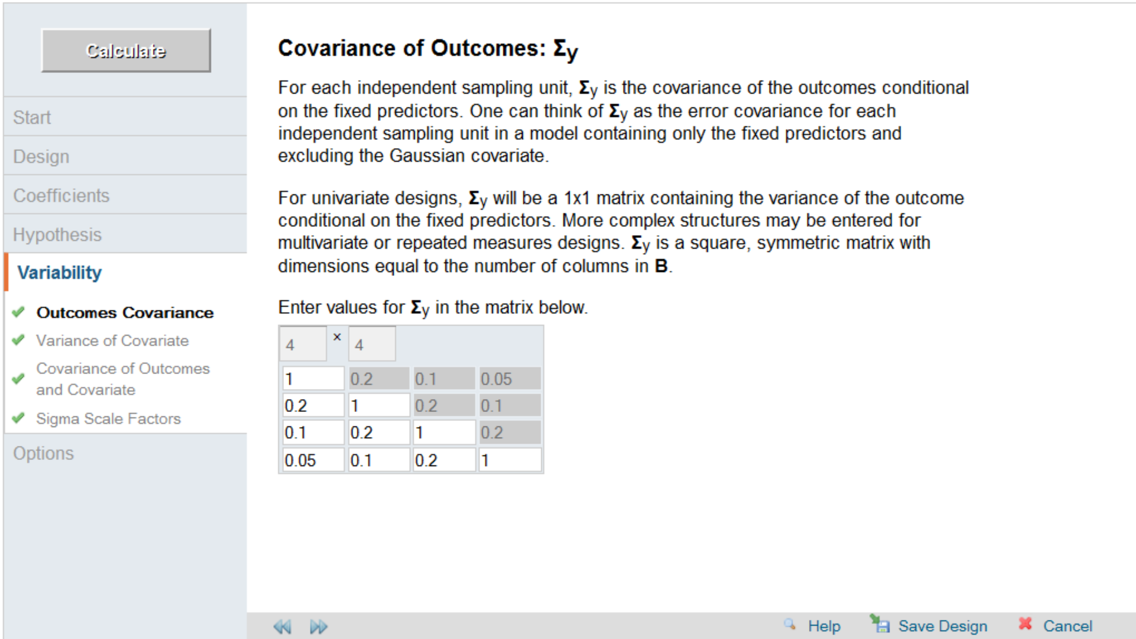

Figure 54: Outcomes Covariance

When finished, click to proceed.

4.5.3. Variance of Covariate

For designs with a Gaussian covariate, the covariate is assumed to have a Gaussian distribution with mean zero and variance σ_g^2. The *Variance of Covariate* screen allows the user to enter a value for σ_g^2.

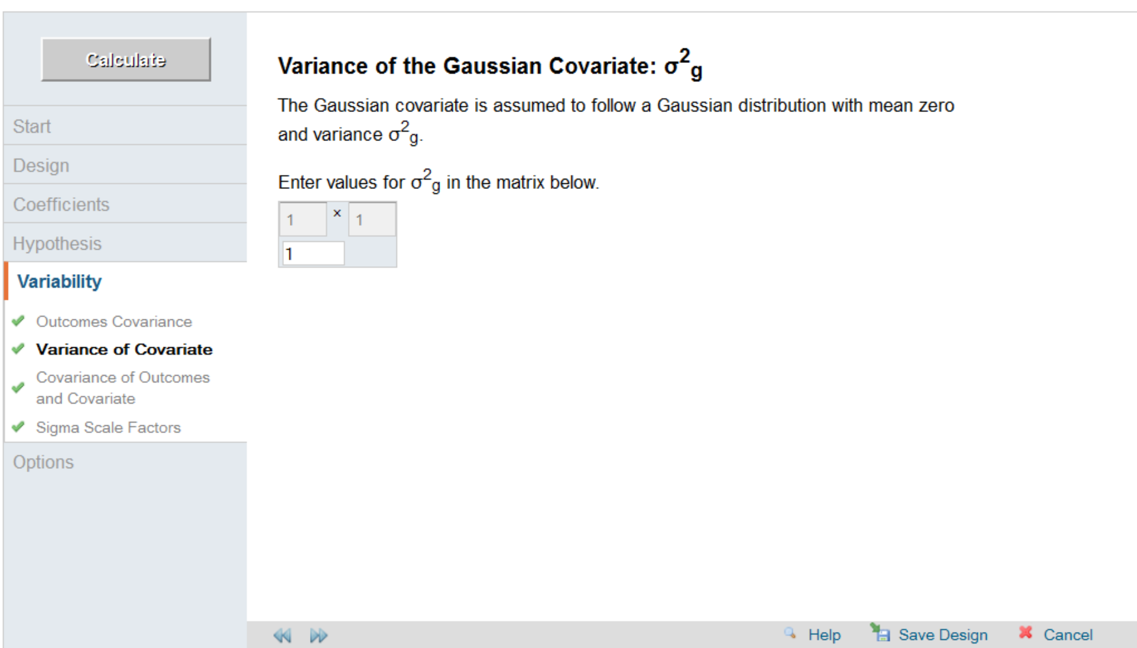

Figure 55: Variance of Covariate

When finished, click ⏩ to proceed.

4.5.4. Covariance of Outcomes and Covariate

When controlling for a Gaussian covariate, power is typically improved when the covariate explains some portion of the variance in the outcome. The covariance matrix between the outcomes and the Gaussian covariate, $\mathbf{\Sigma}_{YG}$, describes the association between the outcomes and the Gaussian covariate. The *Covariance of Outcomes and Covariate* screen allows the user to specify values for $\mathbf{\Sigma}_{YG}$. To ensure conformance with the $\mathbf{\Sigma}_{Y}$ matrix, the dimensions of the $\mathbf{\Sigma}_{YG}$ matrix cannot be modifed on this screen.

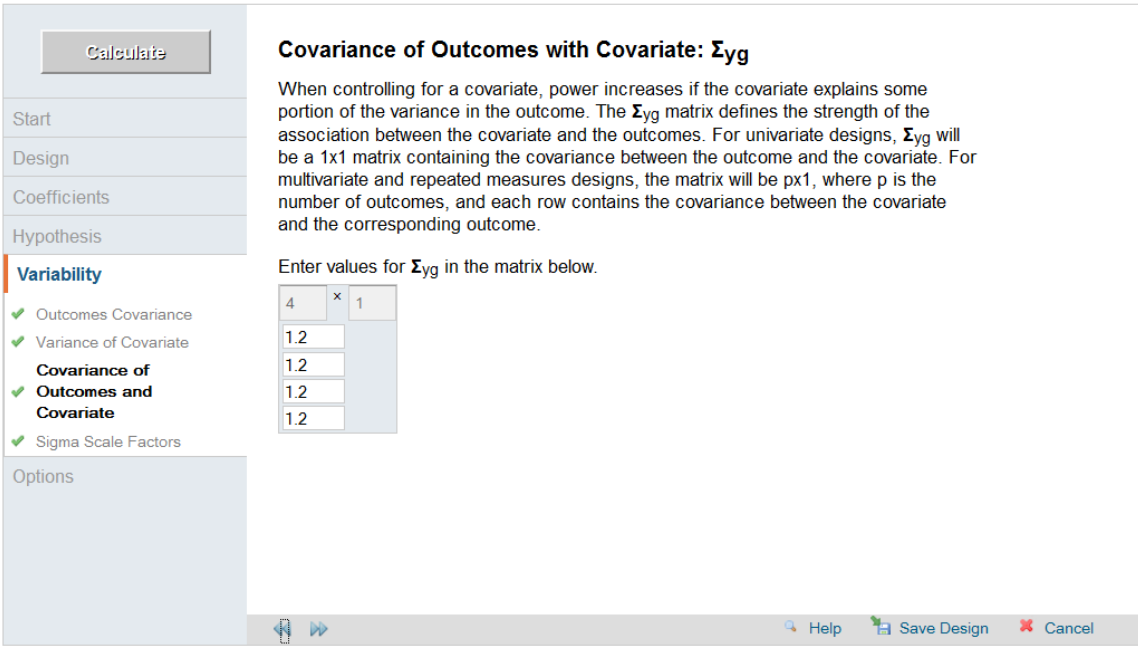

Figure 56: Covariance of Outcomes and Covariate

When finished, click ⏩ to proceed.

4.5.5. Sigma Scale Factors

GLIMMPSE allows users to specify scale factors for the covariance matrices. For the general linear multivariate model with fixed predictors, the scale factors are applied to the user-specified $\mathbf{\Sigma}_e$ matrix. For the general linear multivariate model with fixed predictors and a Gaussian covariate, the scale factors are applied to the $\mathbf{\Sigma}_e$ matrix, which is calculated from $\mathbf{\Sigma}_Y$, $\mathbf{\Sigma}_g$, and $\mathbf{\Sigma}_{Yg}$. Since variability can dramatically impact power, it is common to calculate power for the proposed value, as well as alternative values such as half and twice the proposed value.

To specify one or more covariance scale factors, enter the scale factors in the Σ_E *Matrix Scale Factors* box. After each entry, click Add, press Enter on the keyboard, or click anywhere on the screen. To delete a value, select the unwanted value and click Delete to remove the value from the list.

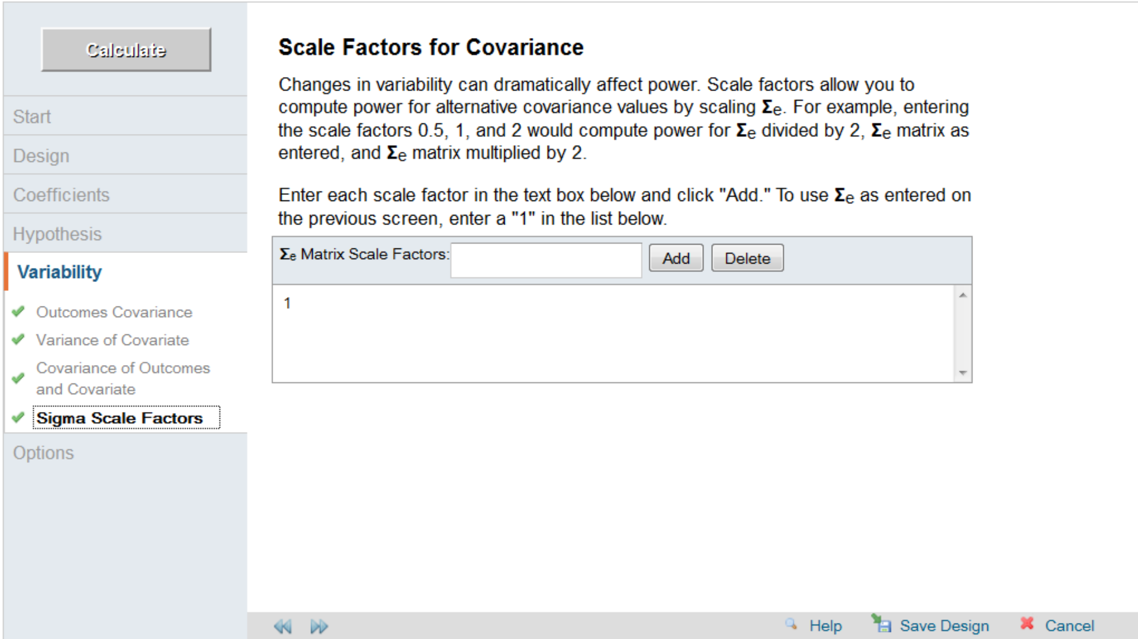

Figure 57: Covariance Scale Factors

When finished, click to proceed.

4.6. Options

This screen provides an introduction to the Options section. After reading the screen, click to proceed.

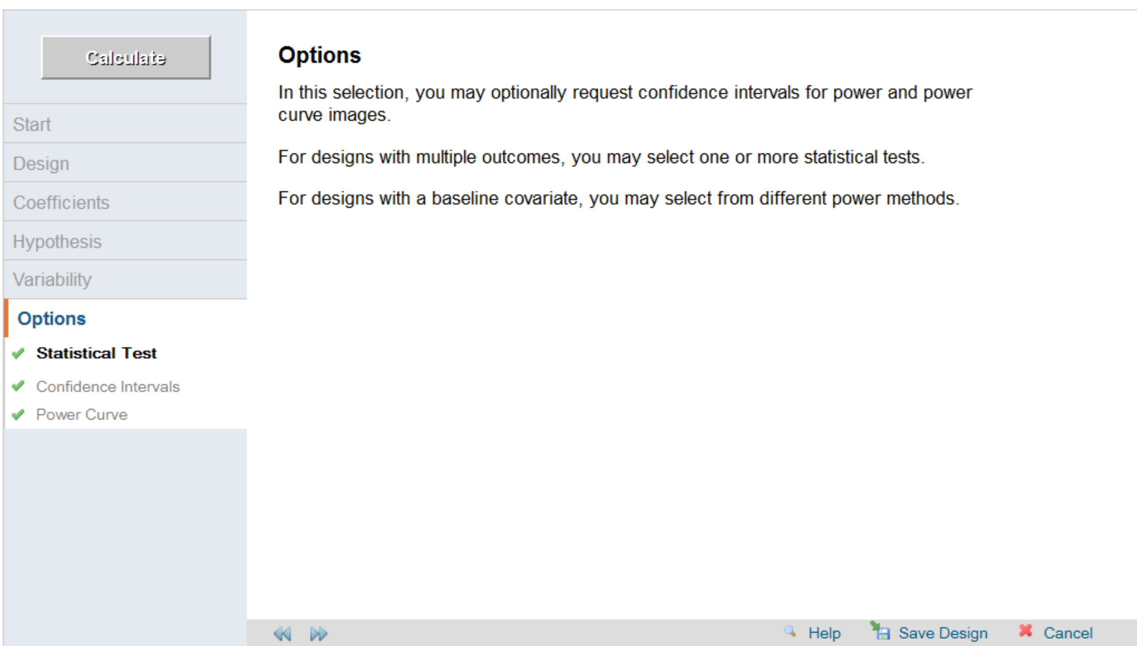

Figure 58: Options

4.6.1. Statistical Tests

The *Statistical Tests* screen allows the user to select one or more statistical tests for the power or sample size calculations. A tutorial providing guidelines for selecting a test is available from the GLIMMPSE Tutorials page at `http://samplesizeshop.org/education/tutorials`. Full theoretical details are available in Muller and Stewart (2006). Select the statistical test(s) by clicking one or more check boxes. For designs with a Gaussian covariate, only the Hotelling-Lawley trace and the Univariate Approach to Repeated Measures are valid.

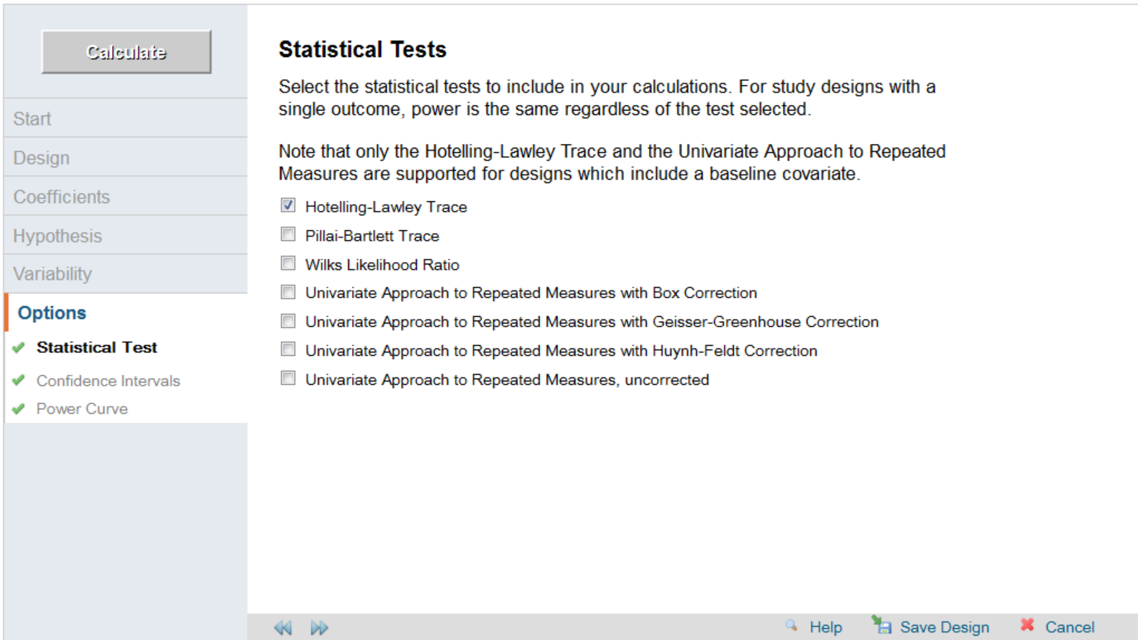

Figure 59: Statistical Tests

When finished, click  to proceed.

4.6.2. Power Calculation Method

For designs with a baseline covariate, two different methods are available to calculate power: quantile and unconditional power. For theoretical details, please see Glueck and Muller (2003). Select the power methods by clicking the checkboxes. If quantile power is selected, the user must also specify one or more quantile values. For example, median power would be obtained by selecting *Quantile* and entering *0.5* in the *Quantiles* list box.

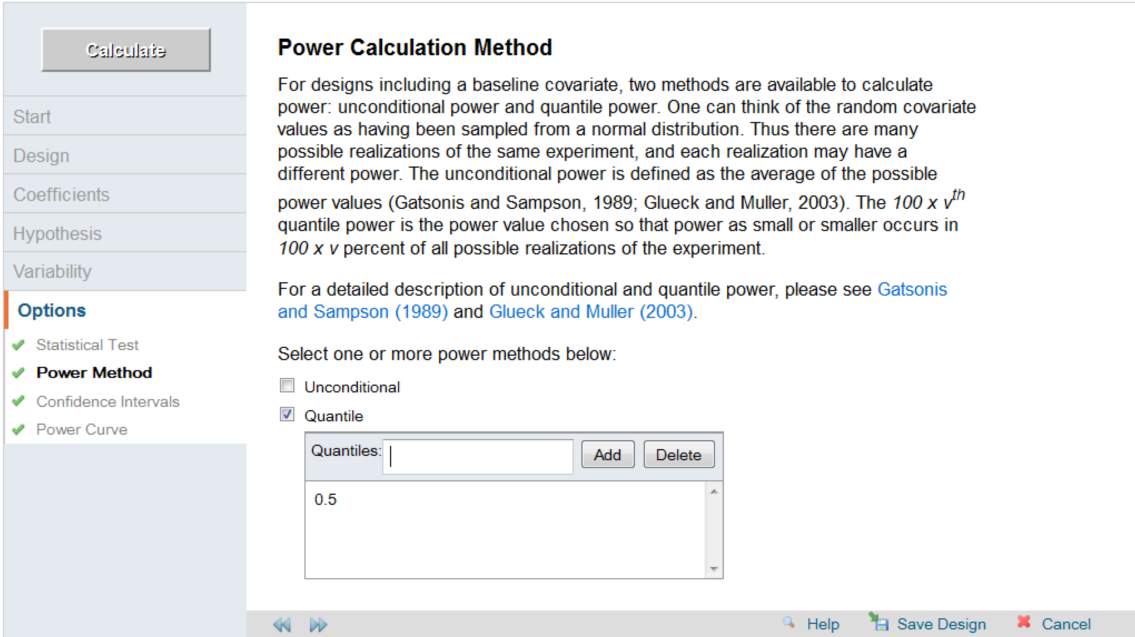

Figure 60: Power Method

When finished, click to proceed.

4.6.3. Confidence Intervals

Power analysis involves some uncertainty in the choices for means and variability. Therefore, the *Confidence Intervals* screen allows the user to request confidence intervals on the power results. To include confidence intervals, uncheck the checkbox. The information on the confidence interval screen describes the data set (or publication) from which the choices for means and variances were obtained. For example, if a scientist was calculating power based on the means and variances obtained from pilot data, the scientist would enter information describing the pilot data set. The following information is required:

The *Assumptions* section allows the user to indicate if he or she is uncertain about the variance, but reasonably certain of the mean values, or uncertain of both the means and variance.

The *Upper and lower tail probabilities* define the width of the confidence interval. For example, a centered 95% confidence interval would have both upper and lower tail probabilities of 0.025.

The *Total sample size* value indicates the number of independent sampling units in the pilot data set (or publication).

The *Rank of the design matrix* describes a property of the predictor matrix used in the pilot data set. Please see Muller and Stewart (2006) for details about matrix rank.

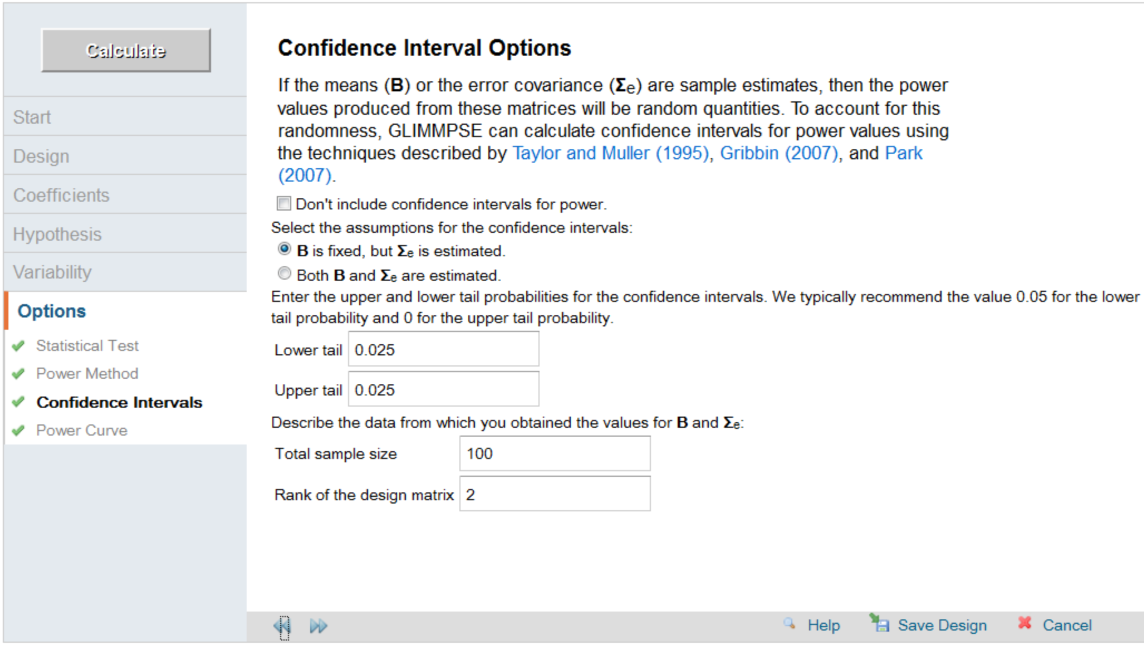

Figure 61: Confidence Intervals

When finished, Click to proceed.

4.6.4. Power Curve Options

The *Power Curve Options* screen allows the user to create a power curve. A power curve describes the change in power (Y axis of the power curve) relative to the total sample size, regression coefficient scale factor, or the variability scale factor (all options for the X axis of the power curve).

To create a power curve, the user must 1) uncheck the check box, 2) select the value to appear on the horizontal axis, and 3) add one or more data series.

Depending on the study design, the user may request a large number of power or sample size values in a single request. A data series is defined by selecting a subset of the power or sample size values. The user creates a data series by selecting values for several study design variables and clicking the Add button. A data series will be displayed as a single line on the power curve plot.

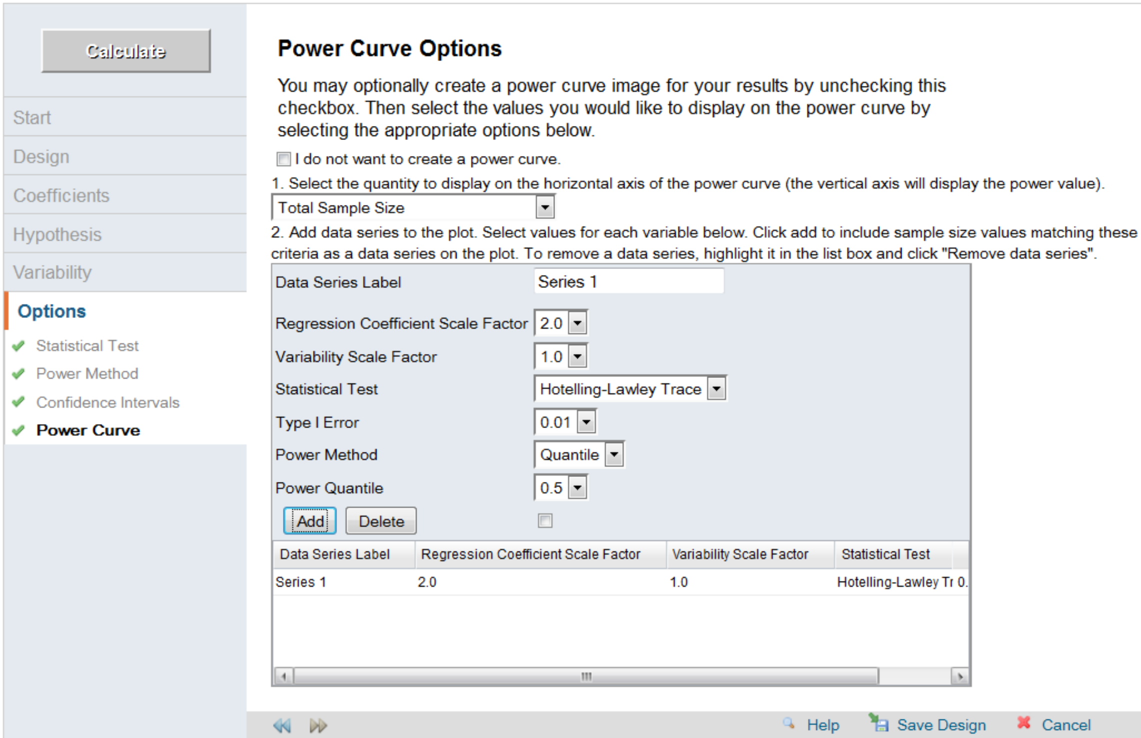

Figure 62: Power Curve

When finished, click to proceed to the results screen. Note that the *Power Curve Options* screen is the final screen in the GLIMMPSE wizard. If the study design is not complete, the button will be disabled.

4.7. Calculate

When sufficient information has been entered for your power or sample size calculation, the *Calculate* button will be highlighted green. Click Calculate to receive the results of your power analysis. Example results are shown in Figure 63. For detailed information regarding the Power Results table, refer to Table 1.

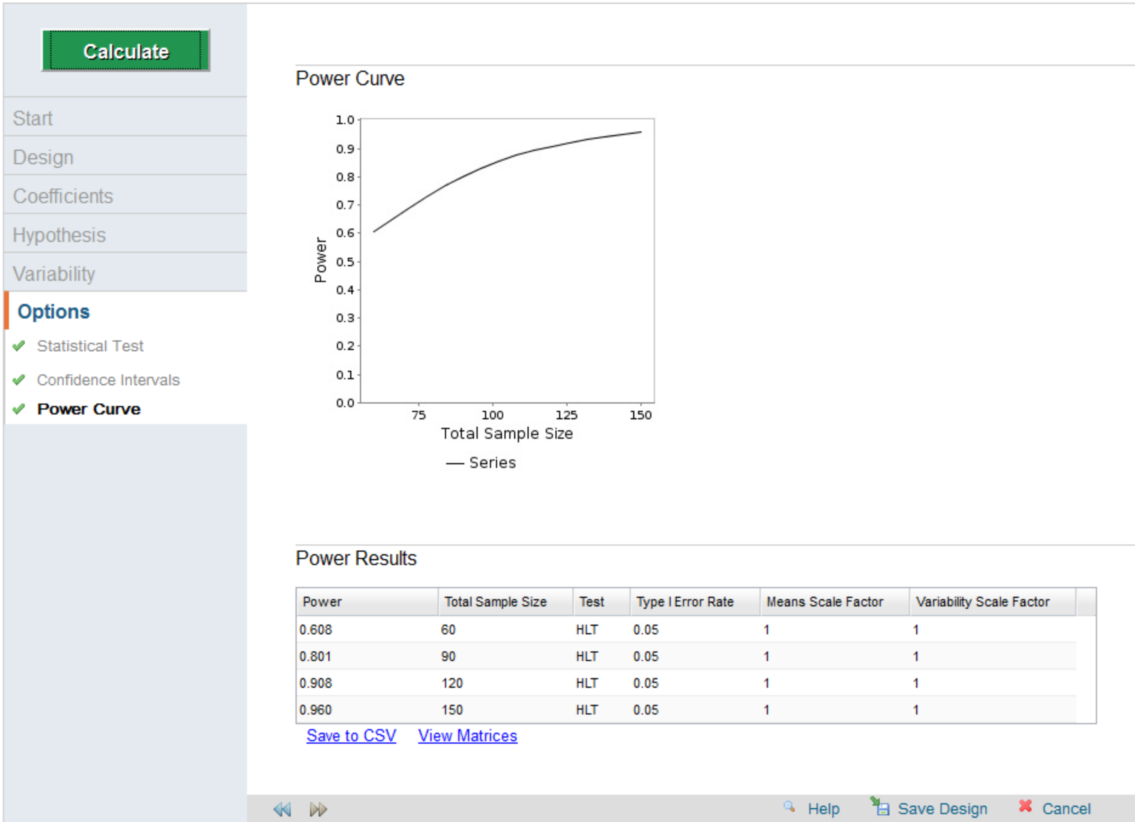

Figure 63: Results

5. Additional GLIMMPSE Resources

Additional resources for GLIMMPSE are available at http://samplesizeshop.org. The Sample Size Shop project is a collaborative effort between the University of Florida and the University of Colorado Denver. The goals of the project are to develop new statistical methods for calculating power and sample size, provide user friendly software to perform the power and sample size calculations, and educate researchers regarding both the methods and the software. The following online resources are available for the GLIMMPSE software.

1. Tutorials demonstrating power and sample size calculations with GLIMMPSE for a variety of study designs are available at http://samplesizeshop.org/education/tutorials
2. Validation reports showing the accuracy of GLIMMPSE calculations are available at http://samplesizeshop.org/documentation/glimmpse/glimmpse-validation-results-2/
3. Technical documentation for the software is available at http://samplesizeshop.org/documentation/glimmpse/
4. GLIMMPSE software modules are available for downloaded from http://samplesizeshop.org/software-downloads/glimmpse-software-downloads/

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